



# Patterns in Nature 10 Summary

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# Pattern formation in the human world

- Well-informed leader
- Building by blueprint
- Following a recipe
- Templates

Usually, several of these mechanisms interact.

The instructions are “external”.

# Two types of waves

## „Normal“ waves

- Mechanism:  
restoring force
- Circular shapes
- Interaction
  - Superposition
  - Diffraction



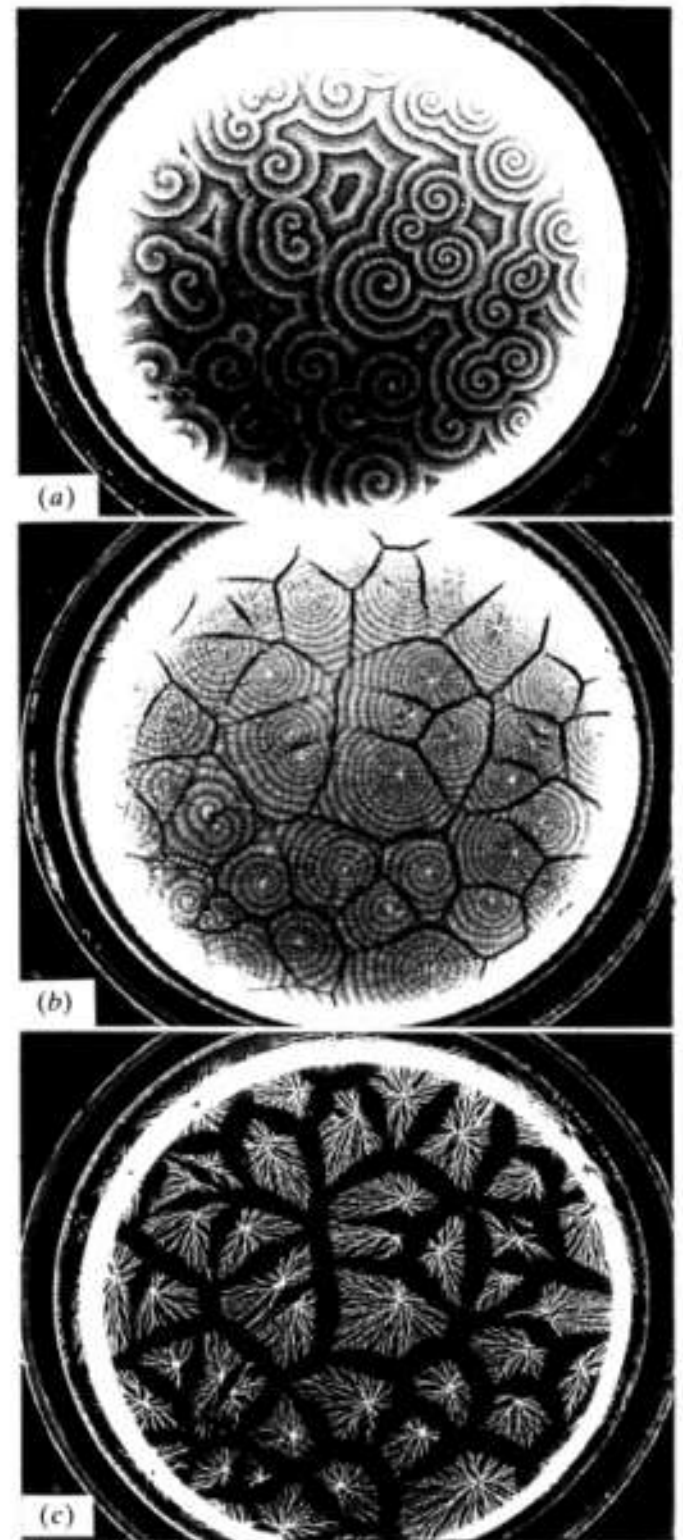
## Excitation waves

- Mechanism:  
excitation/latency
- Spiral shapes
- Interaction
  - Extinction

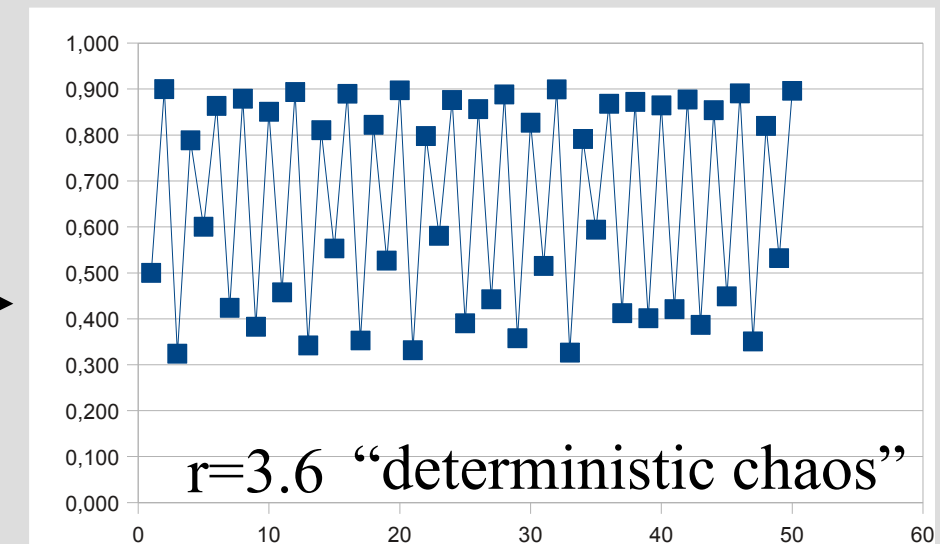
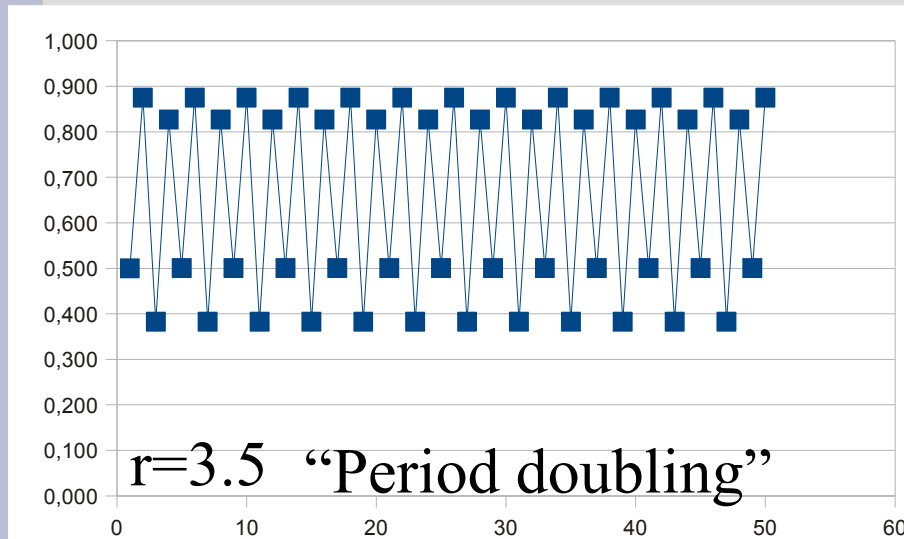
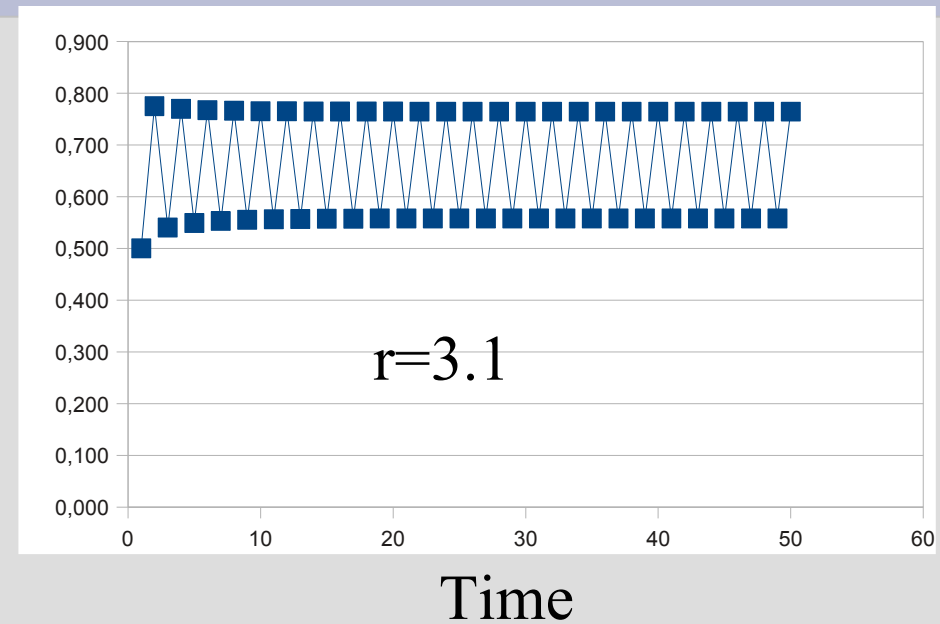
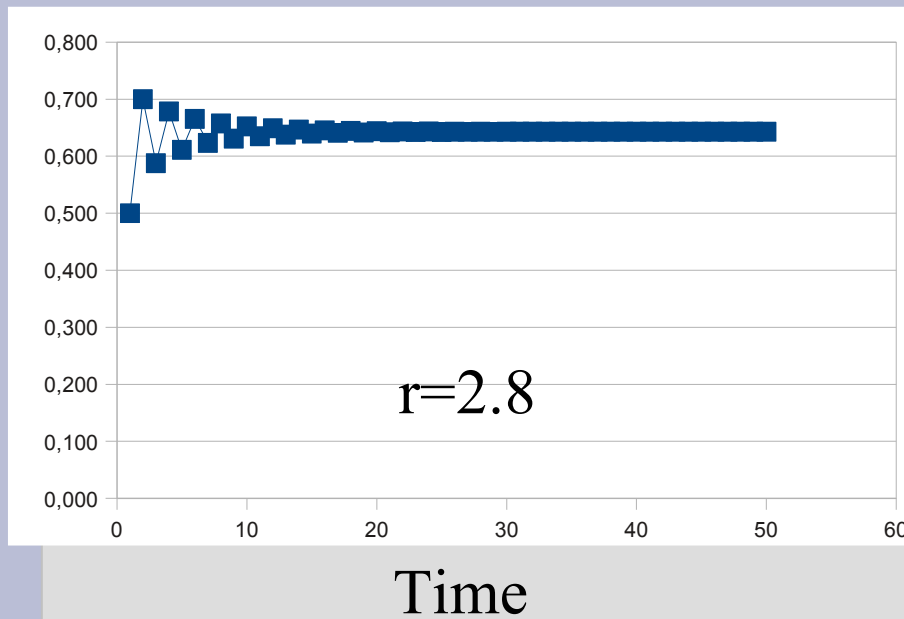


# Excitation waves in slime mold

- cAMP: cyclic adenosine monophosphate
- Spiral waves of cAMP induce:
  - (a) Cell movement
  - (b) onset of cell streaming
  - (c) development of stream morphology



# The logistic map (modelled in a spreadsheet)



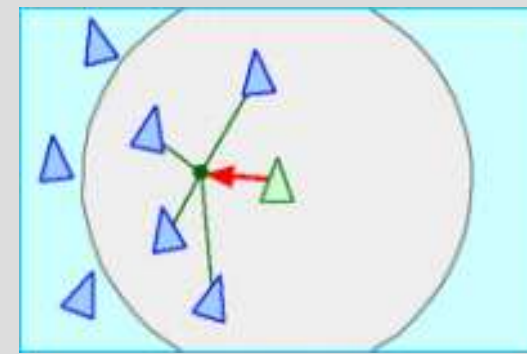
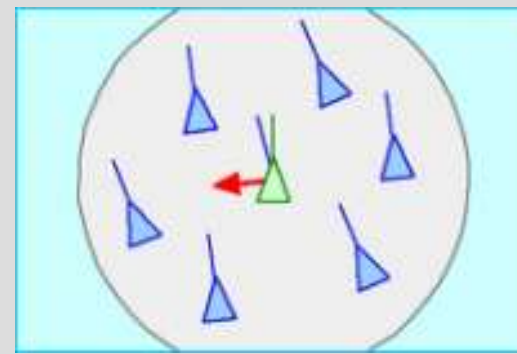
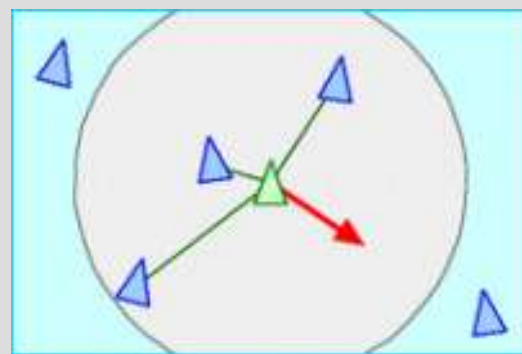
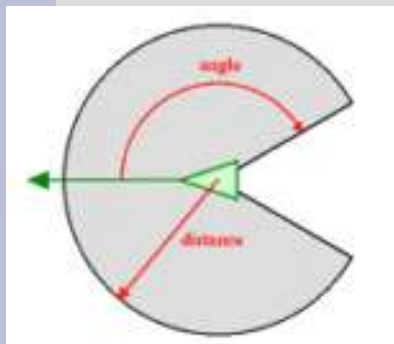
# Deterministic Chaos

- A system that follows deterministic rules (and therefore is – in principle – completely predictable)
- But it is very sensitive to the initial conditions: small changes alter the result completely (Nonlinearity)
- Therefore, in practice it is not predictable because one can never know all initial conditions with infinite precision.



# Understanding swarm behaviour

- individual-based model developed by Craig Reynolds (1986)
- “Boids” (elementary “animals”):
  - they react only to their local neighbourhood
  - neighbourhood characterised by distance & angle
  - they follow 3 simple behaviour rules



A Boid's  
neighbourhood

**Separation**

steer to avoid crowding  
local flockmates

**Alignment**

steer towards the  
average heading  
of local flockmates

**Cohesion**

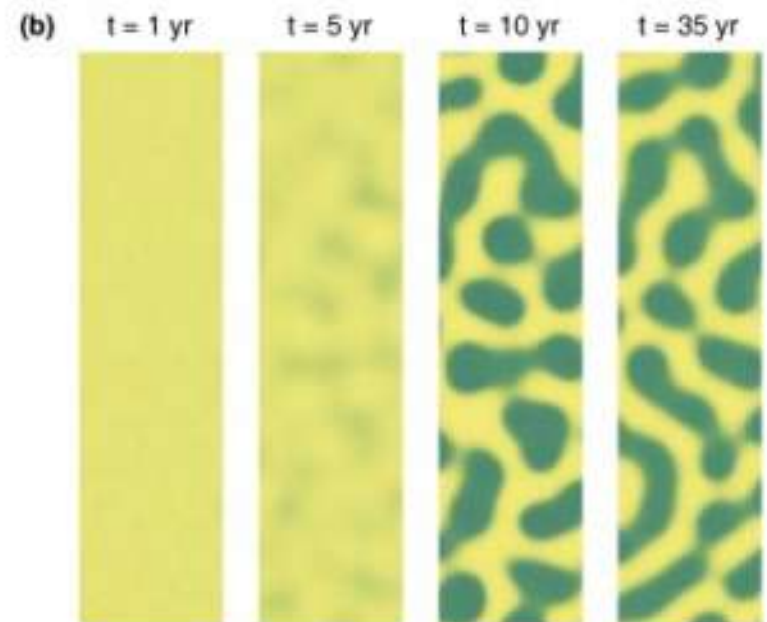
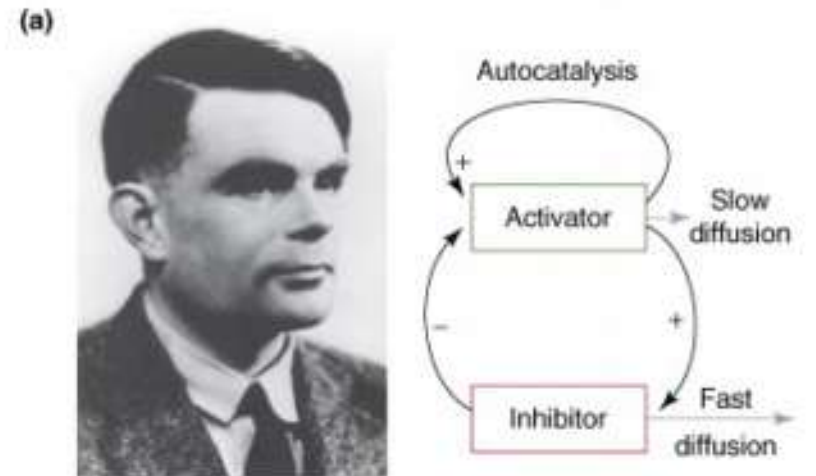
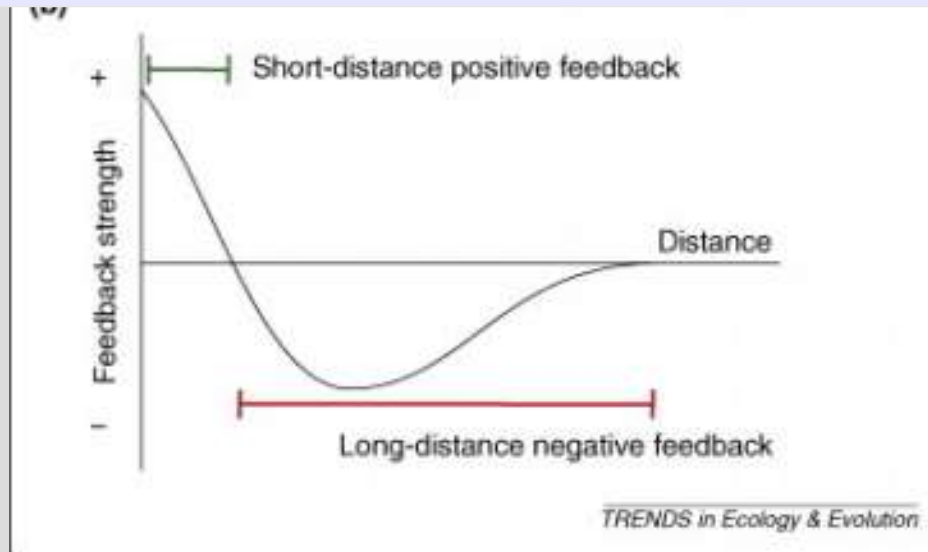
steer to move toward  
the average position  
of local flockmates



# Activator-Inhibitor Model

feedback: part of a system's output influences the input

- positive feedback: the system responds to perturbations in the same direction as the perturbation
- negative feedback: it responds in opposite direction



# Zebras

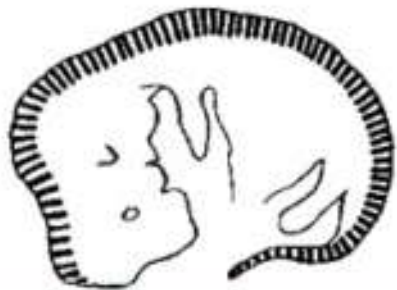
Imperial Zebra: week 5

Mountain Zebra: week 4

A

B

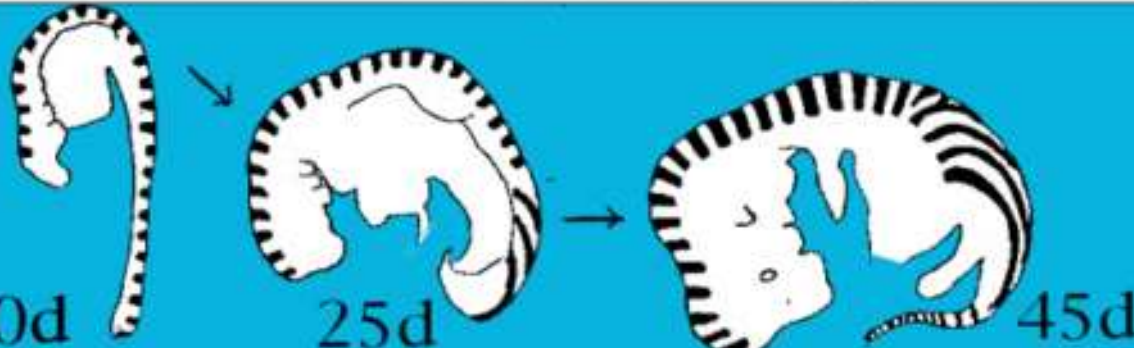
A



B



C



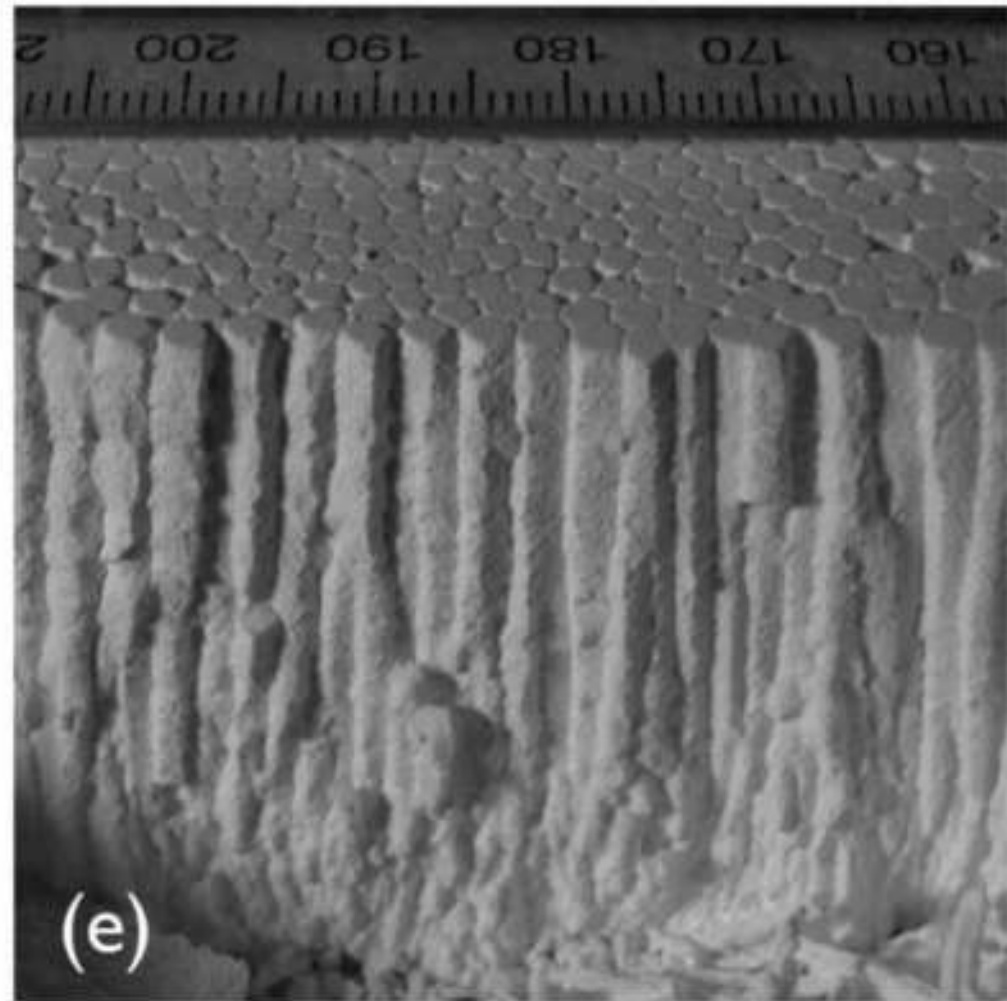
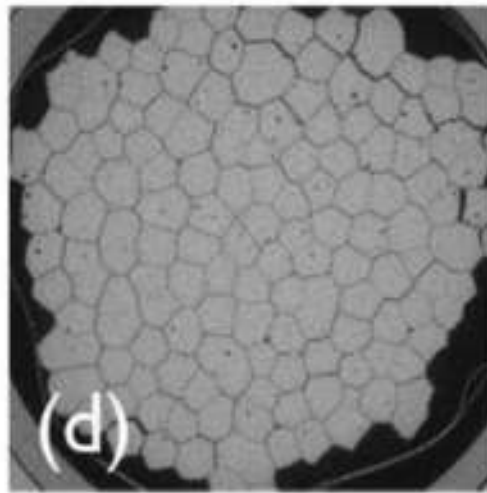
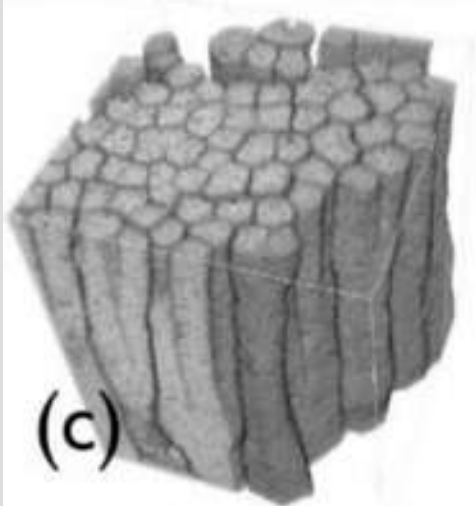
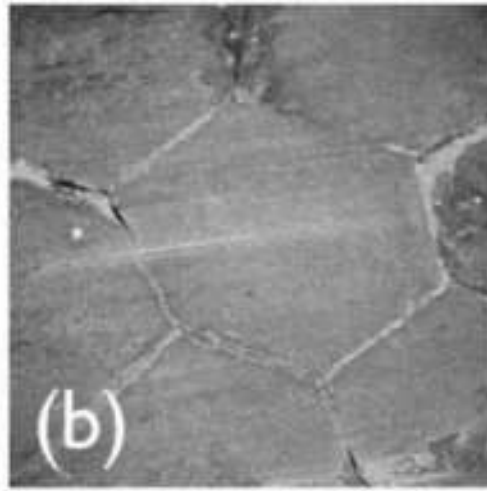
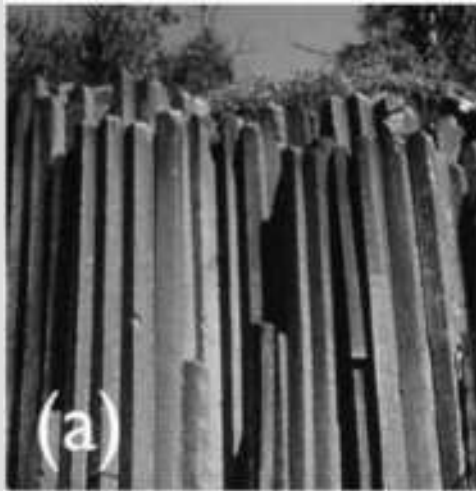
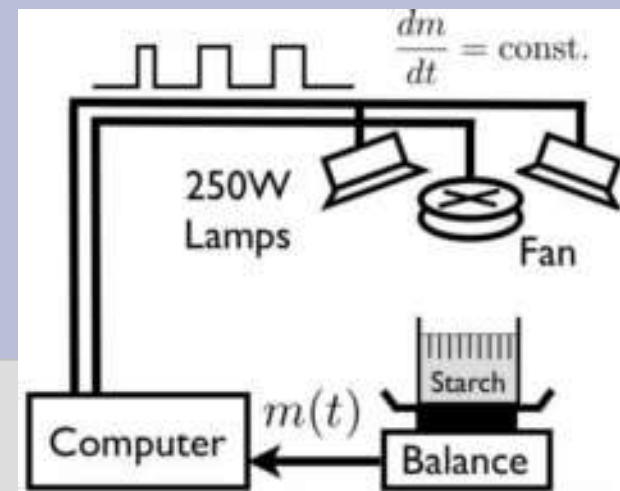
Common Zebra: week 3

C

D

- (a) Imperial zebra (*Equus grevyi*)
- (b) Mountain zebra (*Equus zebra*)
- (c) Common zebra (*Equus burchelli*)
- (d) Quagga (*Equus quagga*).

# An experiment with cornstarch

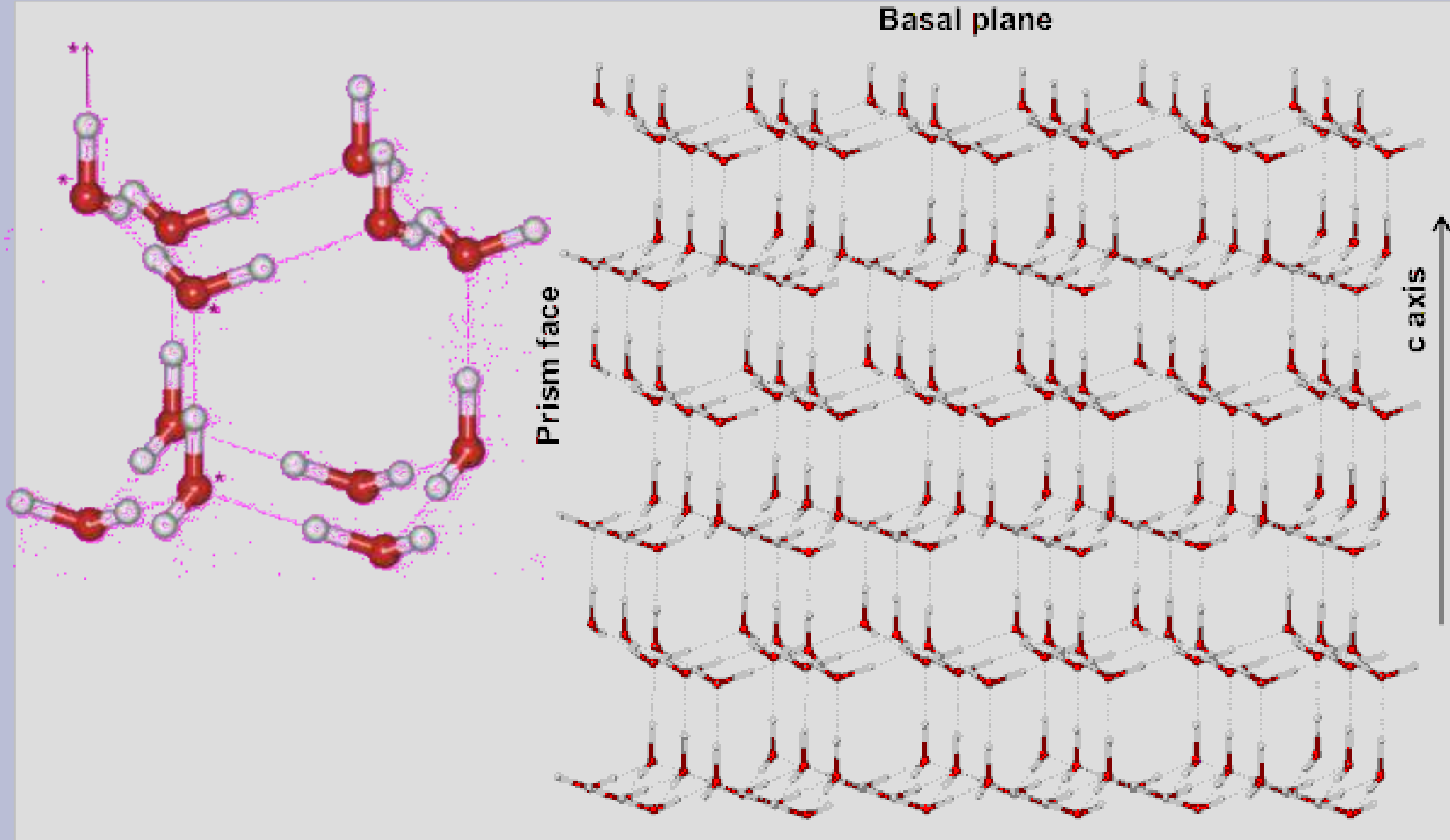


# Snowflakes

Wilson Bentley (1865-1931)



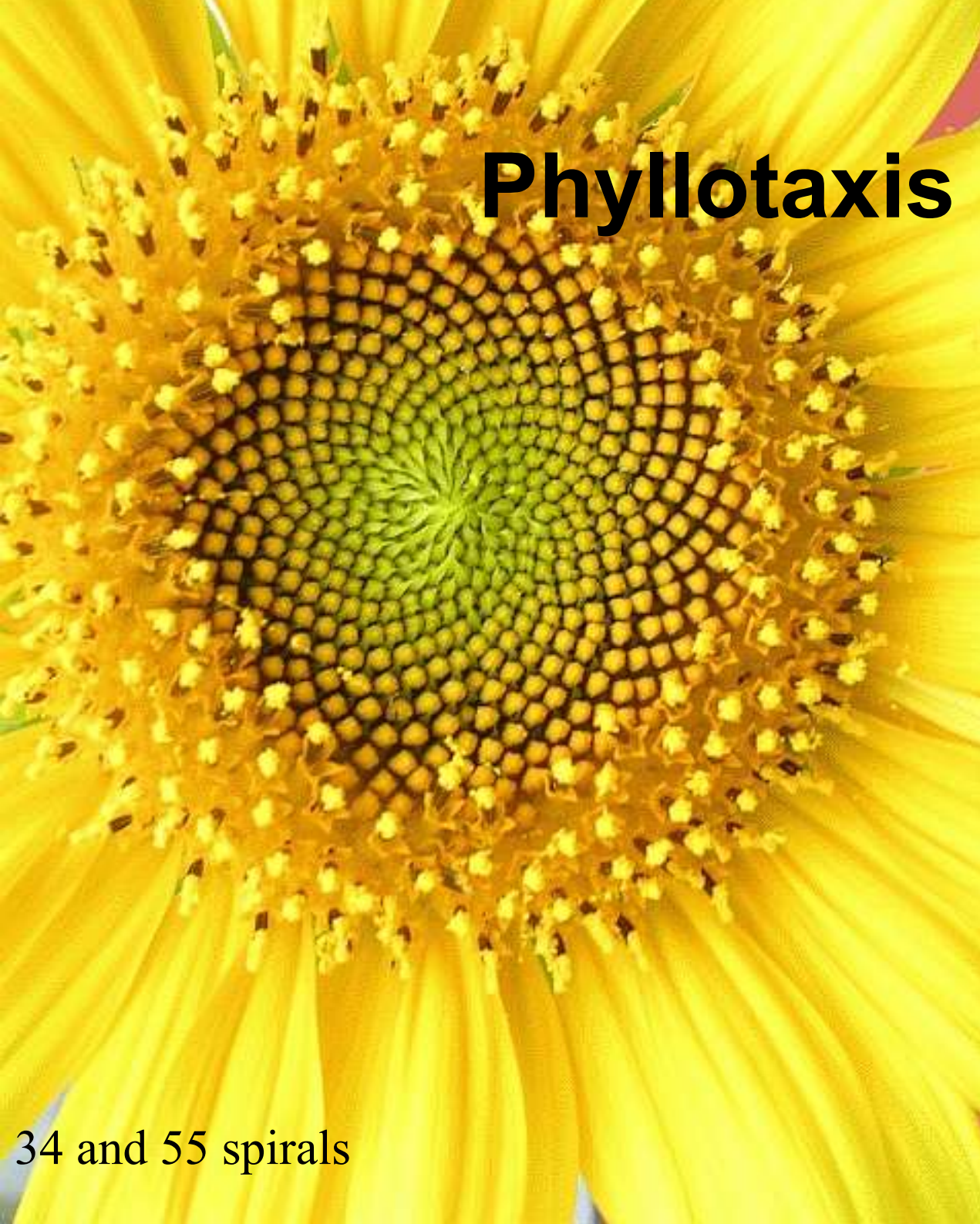
# Water Ice (hexagonal Ice $I_h$ )



# Other fingering mechanisms

- Basic idea: Growth at the tip is easier than at the base (positive feedback!)
- A lot of different examples and names:
  - Diffusion limited aggregation (DLA): growth of dendrites in a solution (e.g. snowflakes)
  - Viscuous fingering; flow in porous medium (Saffman-Taylor instability)
  - Fingering in solidification (Mullins-Sekerka instability)

# Phyllotaxis

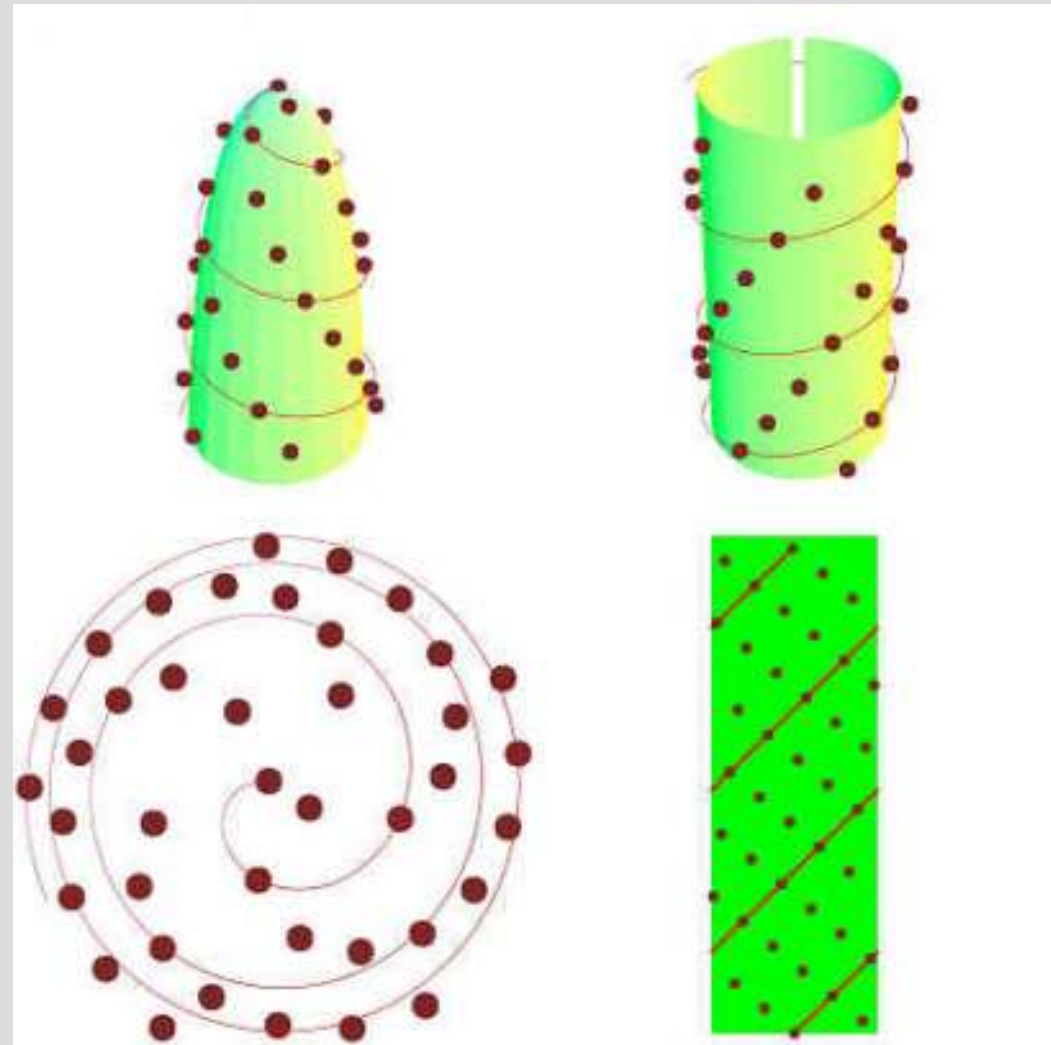
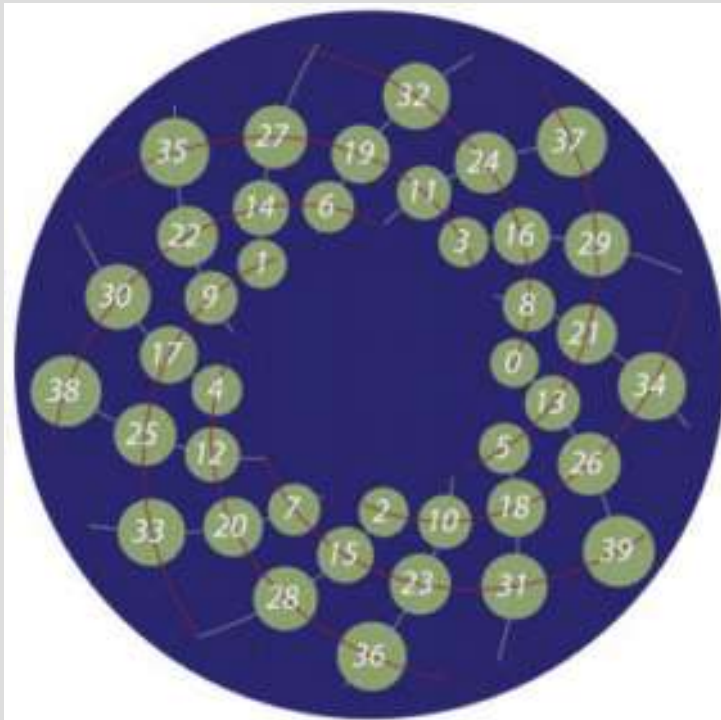


34 and 55 spirals



# Two spirals...

- paristiche: spirals made up of next neighbours
- **generative spiral**





# The most irrational number

There is a „most irrational“ number, and it turns out (surprise, surprise) the golden number.

...

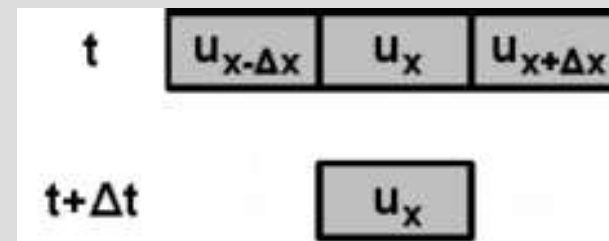
It is the most badly „approximable-by-rational“ number there is!

Its „badness“ is exceeded only by the awkwardness of the preceding sentence.

(Adam, Mathematics in Nature, p220)

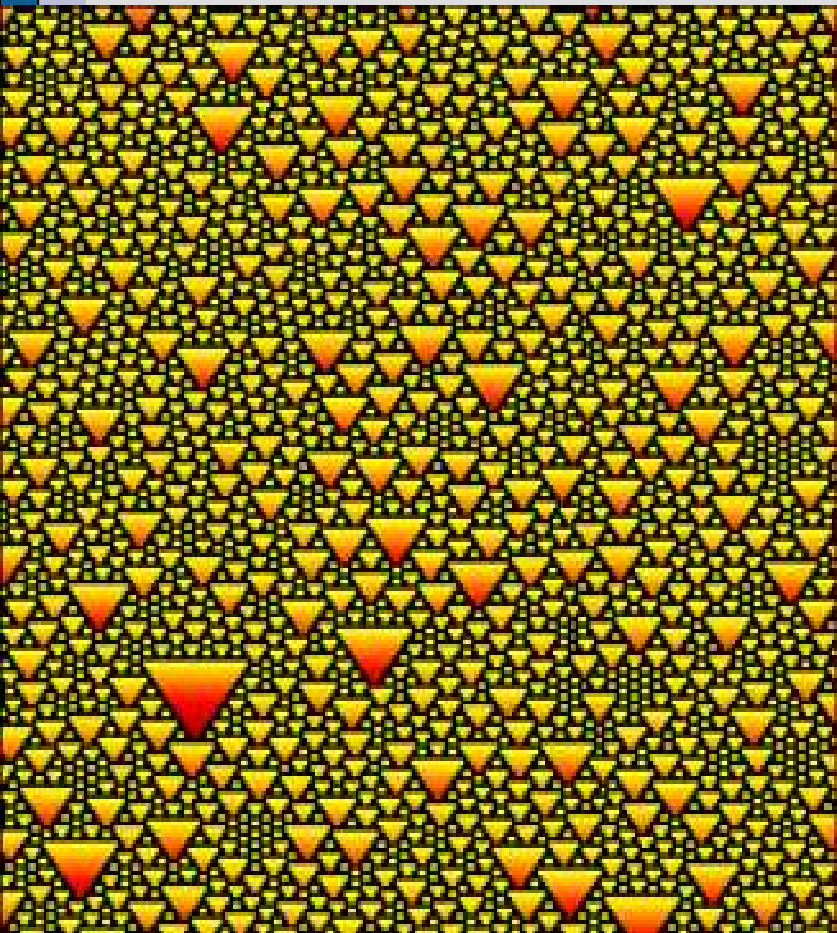
# (1-dimensional) Cellular automata

- Computer model
- The model is “discrete”:
  - Array (line) of cells
  - Each cell can have a finite number of different states (“alive” or “dead”)
- State of a cell changes each timestep depending on the state of its neighbours



# Some more 1-d automata

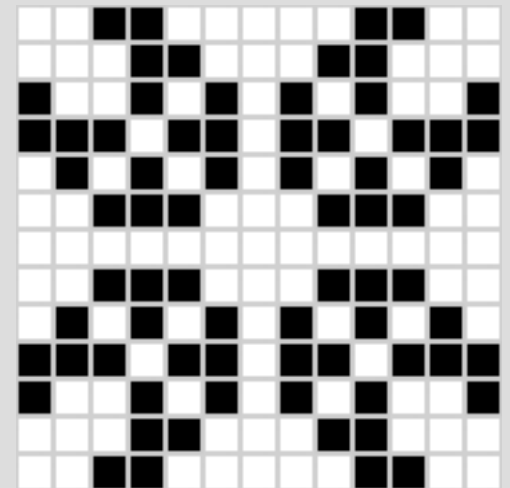
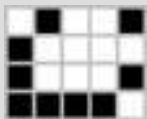
- “Porridge” (R1,C0,M1,S0,S3,B0,B2)



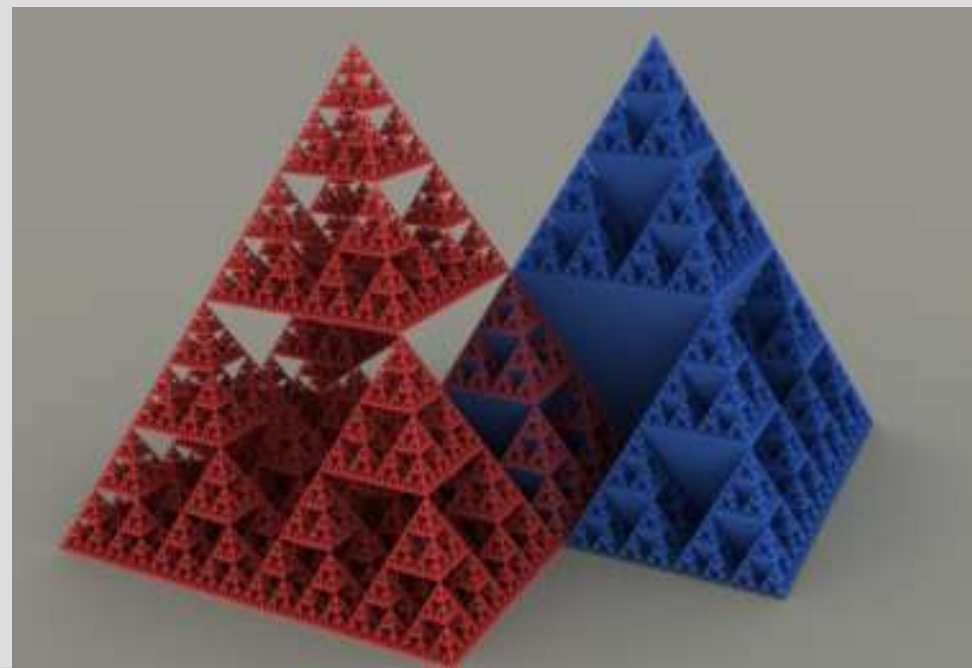
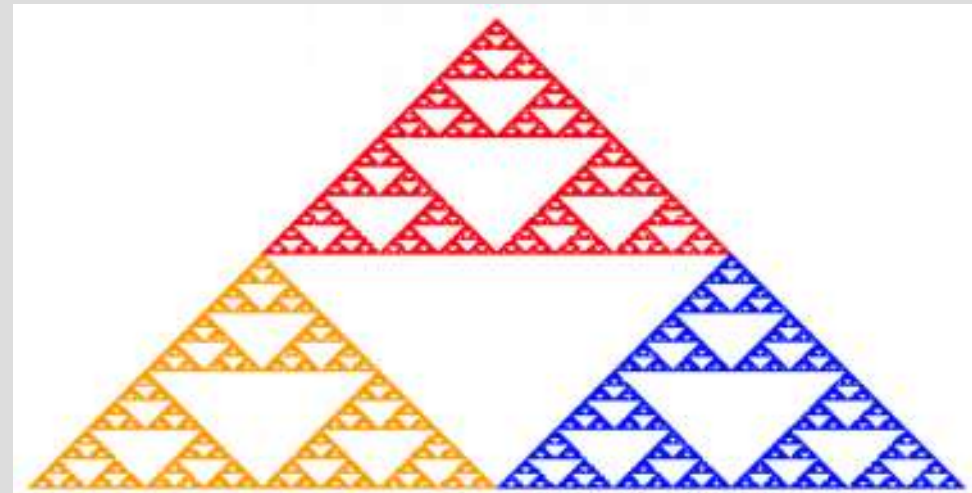
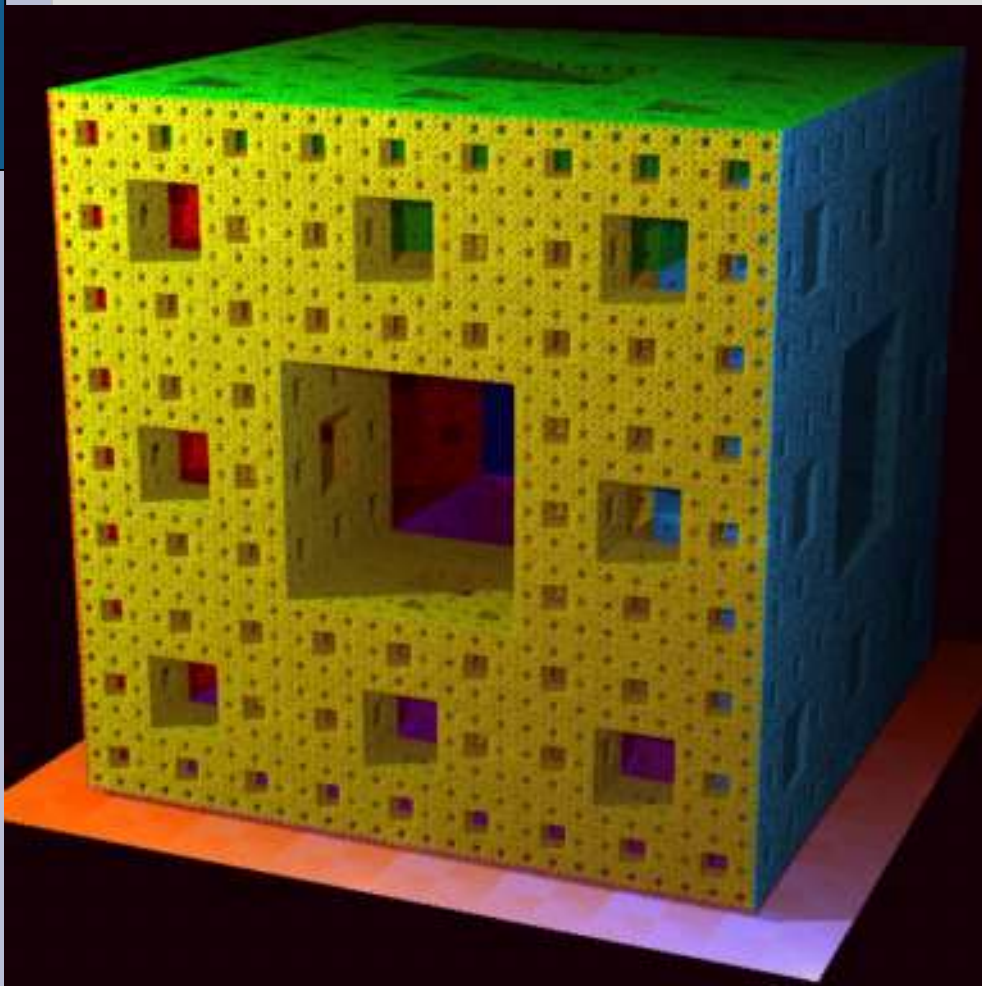
*Conus textile*

# Conway's "Game of Life"

- Still:
  - “Blocks”
  - “boats”
- Stationary 2-phase oscillators:
  - “blinkers”
  - “toads”
- 3-phase oscillator:
  - “pulsar”
- Moving:
  - “gliders”
  - “lightweight spaceship”

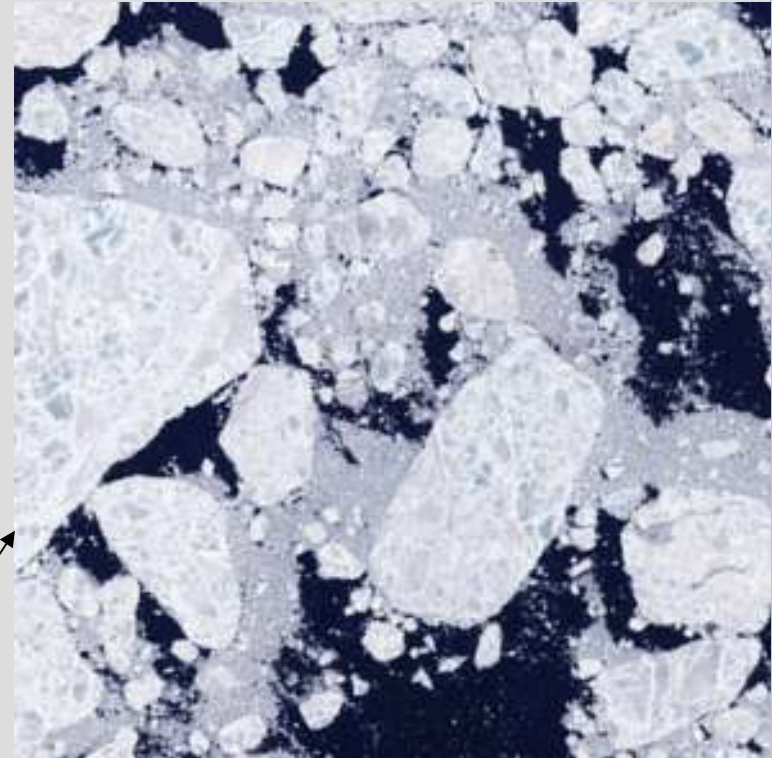
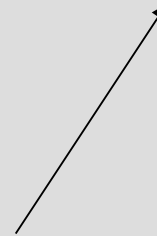
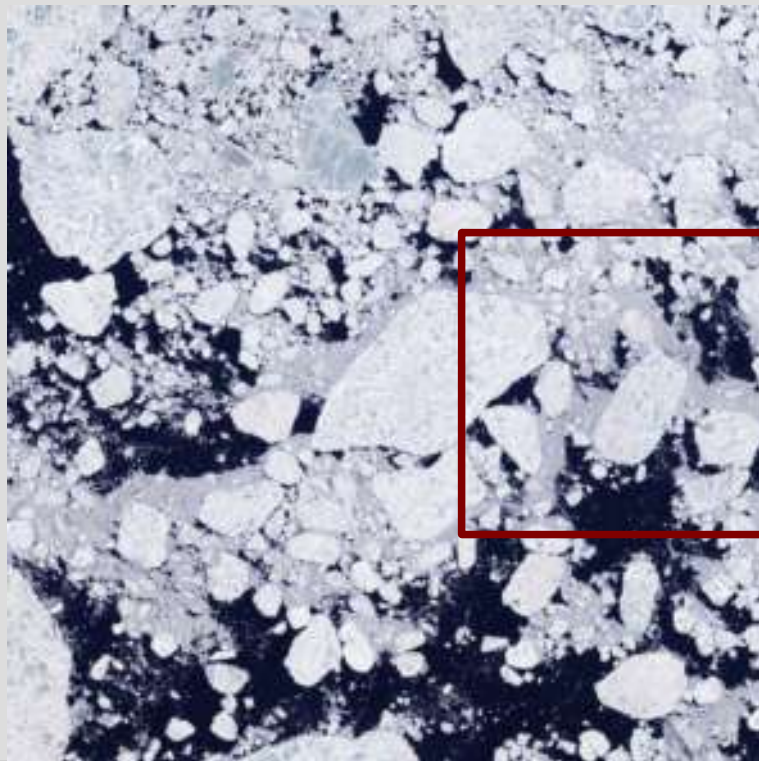
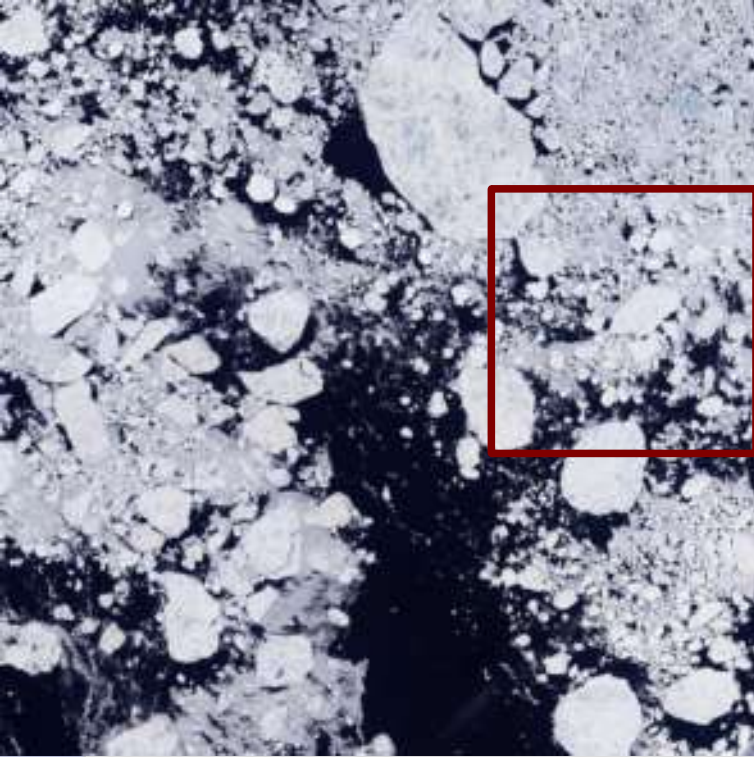


# Menger sponge and Sierpinski pyramid

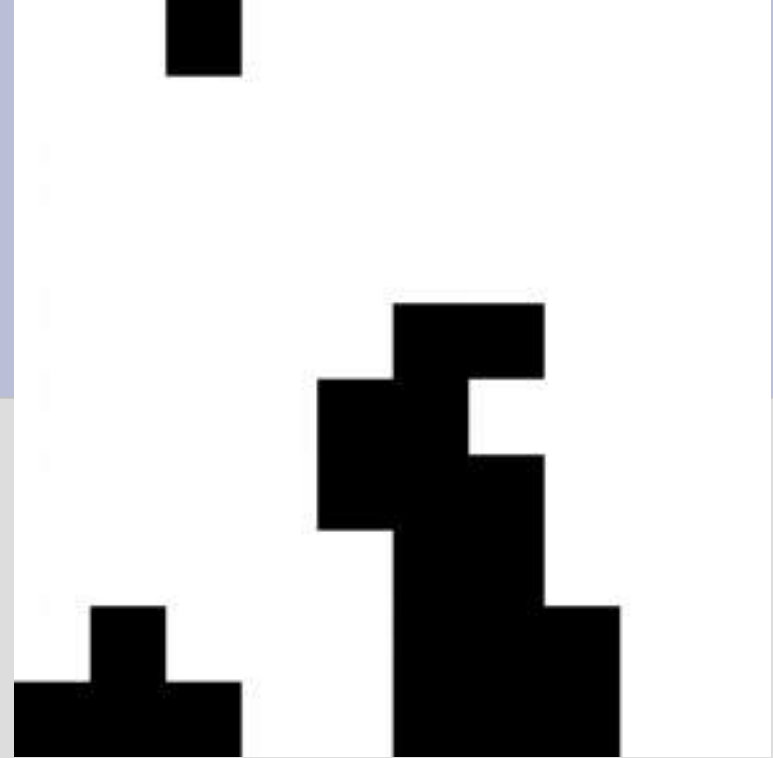
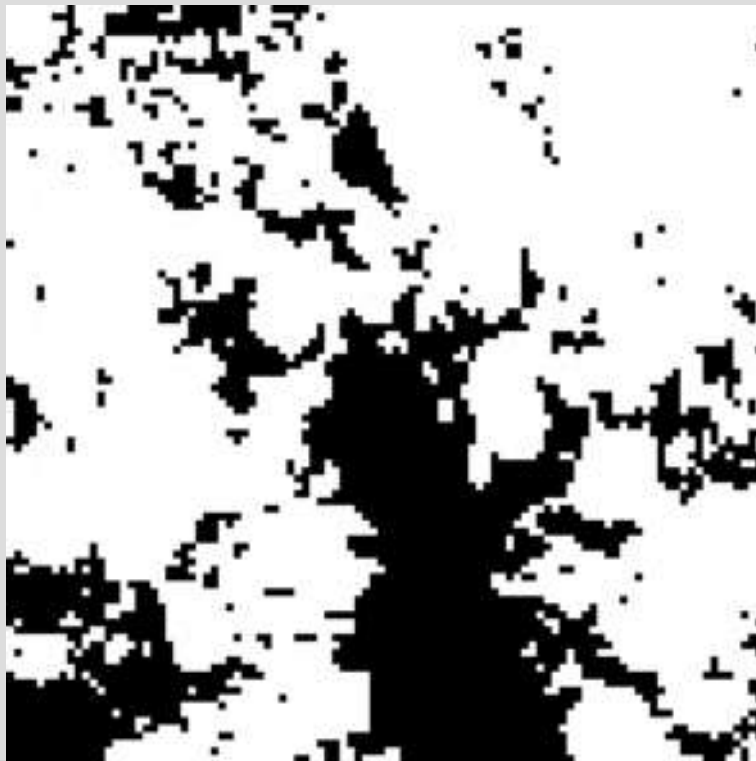
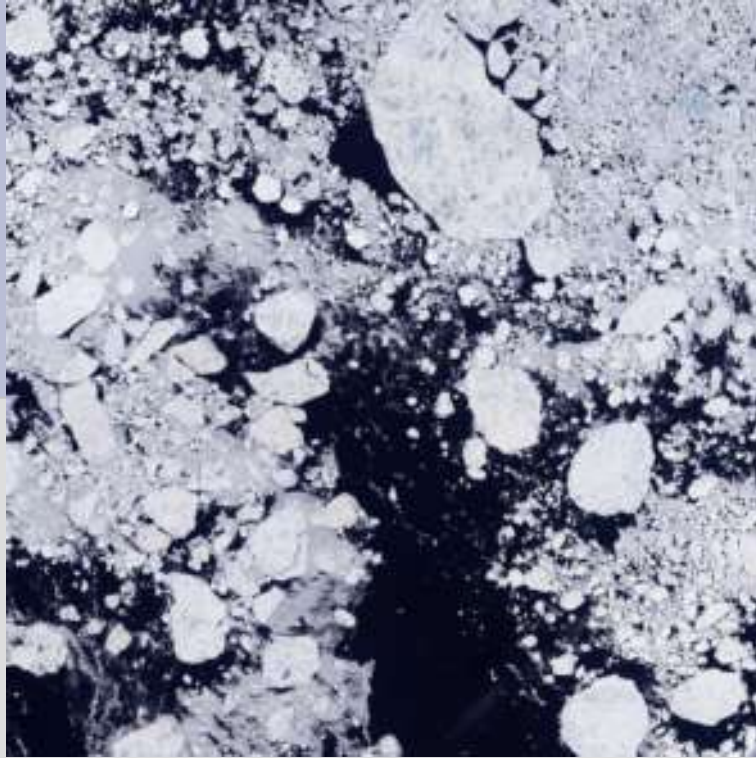




# Self similarity



Full image:  
6,975,486  
/ 10,614,564  
= **65.7%** ice



80 ice pixels out of 100 = **80%** ice cover

Dimension (using the  
10x10 and 100x100 images):

$$a = 10$$

$$N = 7010/80 = 87.625$$

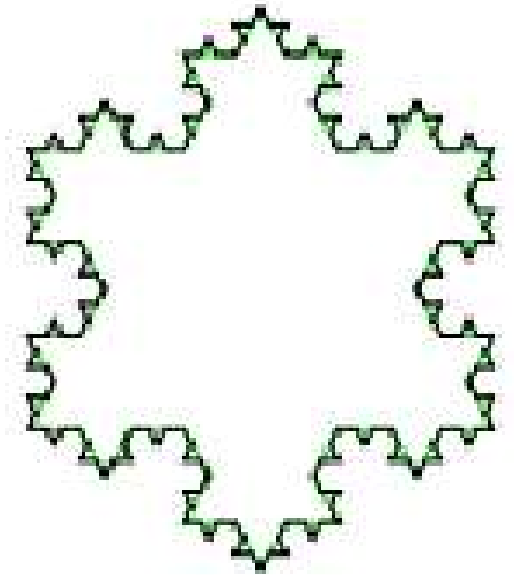
$$\text{dim} = \log(N) / \log(a) = 1.94$$

7010 ice pixels / 10000 in total  
= **70.1%** ice cover



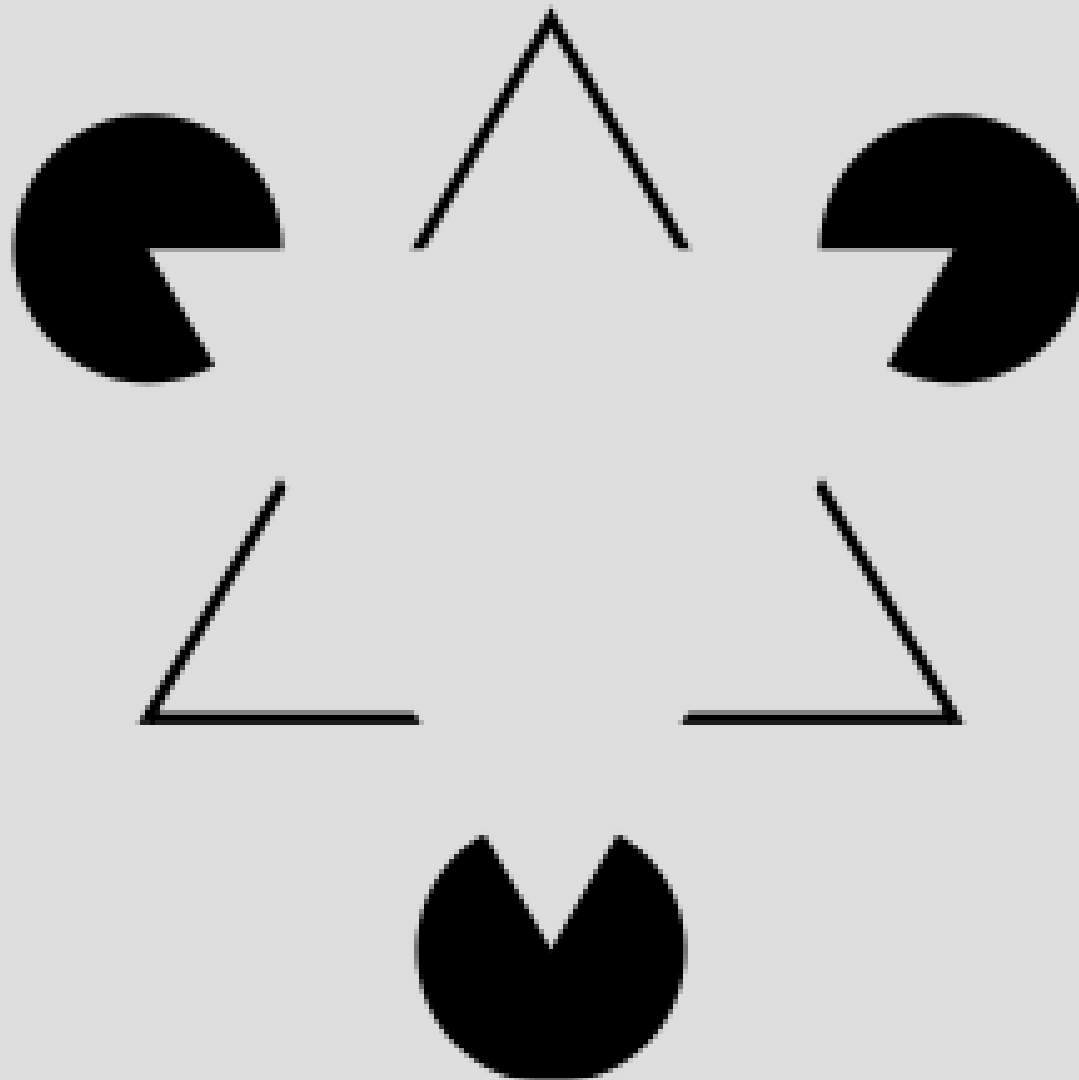
# Frequent features of fractals

- F is **self-similar** (at least approximately or stochastically).
- F has a **fine-structure**: it contains detail at arbitrary small scales.
- F has a simple definition.
- F is obtained through a **recursive** procedure.
- The geometry of F is not easily described in classical terms.
- It is awkward to describe the local geometry of F.
- The size of F is not quantified by the usual measures of length (this leads to the **Hausdorff dimension**)

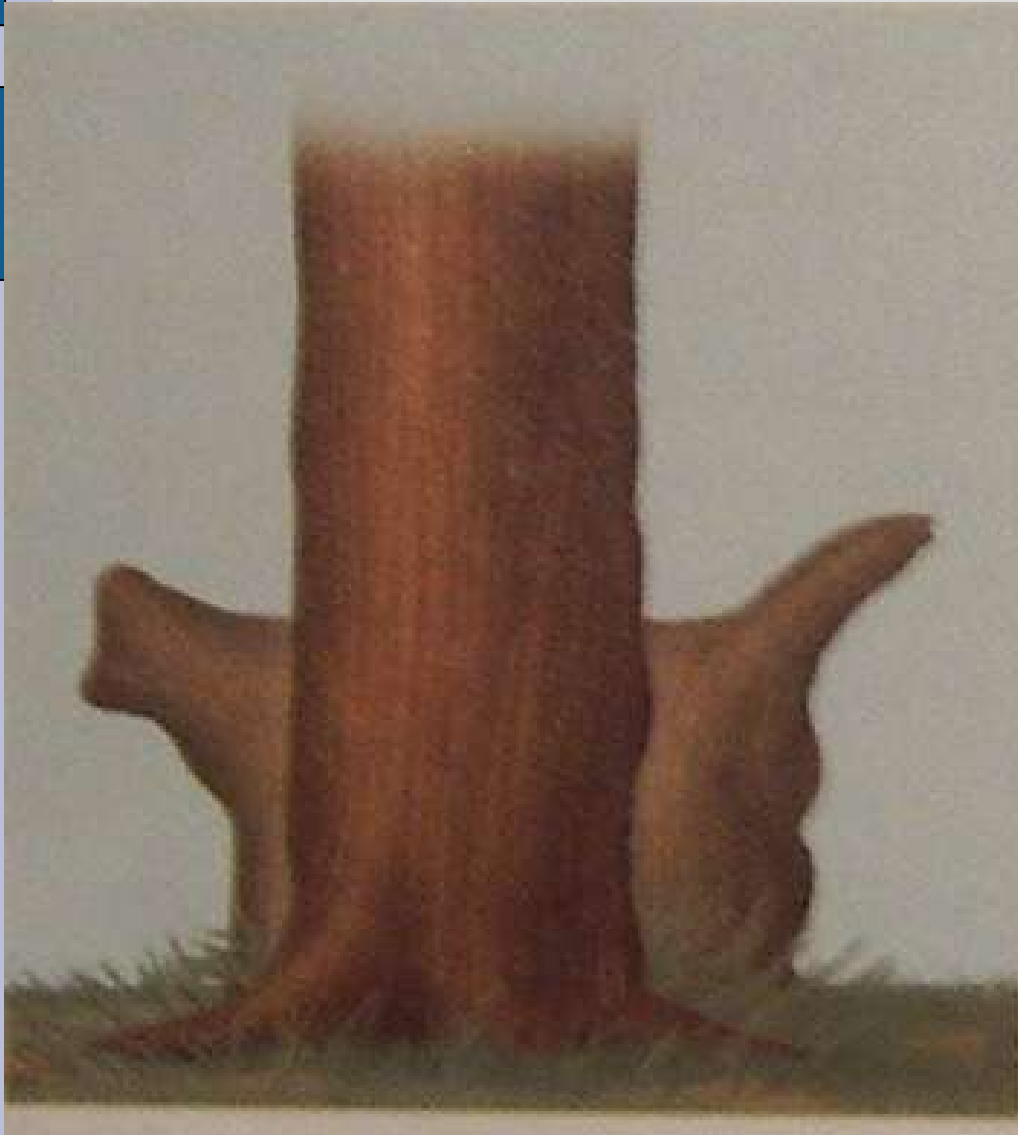


(after Falconer 1990)

# Perception: Kanizsa figures

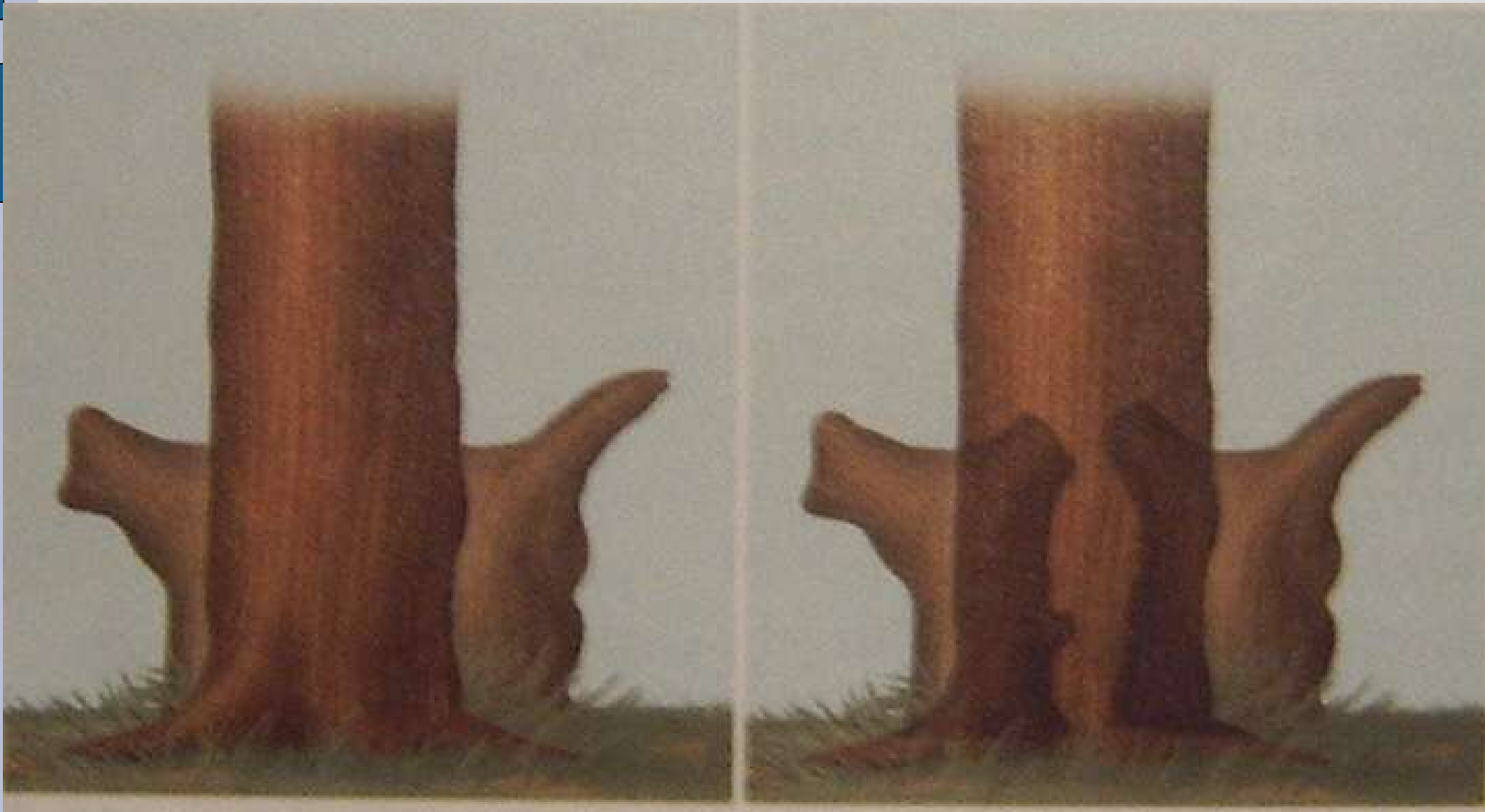


# Gestalt perception



# Why?

## Ambiguity in the real world



# Different levels of description

- Mechanisms:  
BZ reaction, pheromones (social insects), genetics, ...
- Phenomenological description  
excitation wave, fractal, chaos, ...
- Modelling  
cellular automata, boids (swarming behaviour), differential equations, ...

=>

- Need many levels of description
- No simple „one size fits all“ theory

# Self-organization

Self-organization is a process in which pattern at the global level of a system emerges solely from numerous interactions among the lower-level components of the system.

Moreover, the rules specifying interactions among the system's components are executed using only local information, without reference to the global pattern.

(Camazine et al 2001, p. 8)

# Emergence

Lat: *emergere* – to emerge

lit: come or bring to surface/out of the water  
(*ex*: out of; *mergere*: to sink something)

The spontaneous appearance of new features in complex system which cannot (obviously) be reduced to the features of the individual components of the system.

As philosophical concept (mainly in the philosophy of biology), it was developed since the late 19th century

# Aristoteles

The whole is greater than the sum of its parts.

...what is the cause of their unity? In the case of all things which have several parts and in which the totality is not, as it were, a mere heap, but the whole is something beside the parts, there is a cause; for even in bodies contact is the cause of unity in some cases, and in others viscosity or some other such quality. ... What then, is it that makes man one; why is he one and not many, e.g. animal + biped, especially if there are, as some say, an animal-itself and a biped-itself?

Aristotle, *Metaphysics*, VIII:6



# Lewes 1875 coined term „emergent“

Every resultant is either a sum or a difference of the co-operant forces; their sum, when their directions are the same - their difference, when their directions are contrary. Further, every resultant is clearly traceable in its components, because these are homogeneous and commensurable. It is otherwise with emergents, when, instead of adding measurable motion to measurable motion, or things of one kind to other individuals of their kind, there is a co-operation of things of unlike kinds. The emergent is unlike its components insofar as these are incommensurable, and it cannot be reduced to their sum or their difference.

(George Henry Lewes, 1875)

# Ernst Mayr: The Emergence of Evolutionary Novelty (1959)

Ernst Mayr (1904-2005)

- importance of population dynamics
- biological species concept (species not defined by static features, but by the ability to form a breeding community)
- Book: The Emergence of Evolutionary Novelty (1959)

Systems almost always have the peculiarity that the characteristics of the whole cannot (not even in theory) be deduced from the most complete knowledge of the components, taken separately or in other partial combinations. This appearance of new characteristics in wholes has been designated as emergence.

(Ernst Mayr, The Growth of Biological Thought, 1982)

# Konrad Lorenz

## Fulguration

Konrad Lorenz (1903-1989), influential researcher of animal behaviour, criticized the term Emergence, because „emergence“ implies that the arising features were already present, but hidden from sight.

He used Fulguration instead (lat. *fulgur*: lightning flash)

# Goldstein 1999

Emergence is the arising of novel and coherent structures, patterns and properties during the process of self-organization in complex systems.

Common characteristics:

- (1) radical novelty (features not previously observed in systems);
- (2) coherence or correlation (meaning integrated wholes that maintain themselves over some period of time);
- (3) A global or macro "level" (i.e. there is some property of "wholeness");
- (4) it is the product of a dynamical process (it evolves);
- (5) it is "ostensive" (it can be perceived).

# Strong and weak emergence

Weak emergence:

System features can (in principle) be reduced to the interaction of individual components.

Strong emergence:

System features are irreducible, they are qualitatively different from the component level.

Many scientists are very uncomfortable with strong emergence, but it is, for example, widely discussed in the philosophy of mind (e.g. problem of qualia or phenomenal consciousness)

# Questions and difficulties about emergence

The terms patterns, self-organisation and emergence remain elusive. What is meant by „cannot be deduced from the knowledge of the components“ (Mayr) or similar phrases?

- Which features of a complex system are emergent?
- What is meant by „cannot be deduced“?
- Referring to „knowledge“ is awkward; ontological terms should only refer to features of the material world, not our (subjective) knowledge.