

A photograph of a rocky riverbed with a large, intricate ice formation in the center, illustrating natural growth processes. The ice has a complex, branching, and crystalline structure, resembling a snowflake or a frozen splash. The riverbed is composed of many small, smooth, brownish-yellow stones and pebbles. The water is clear, and the background is a bright, overexposed sky.

Patterns in Nature 6

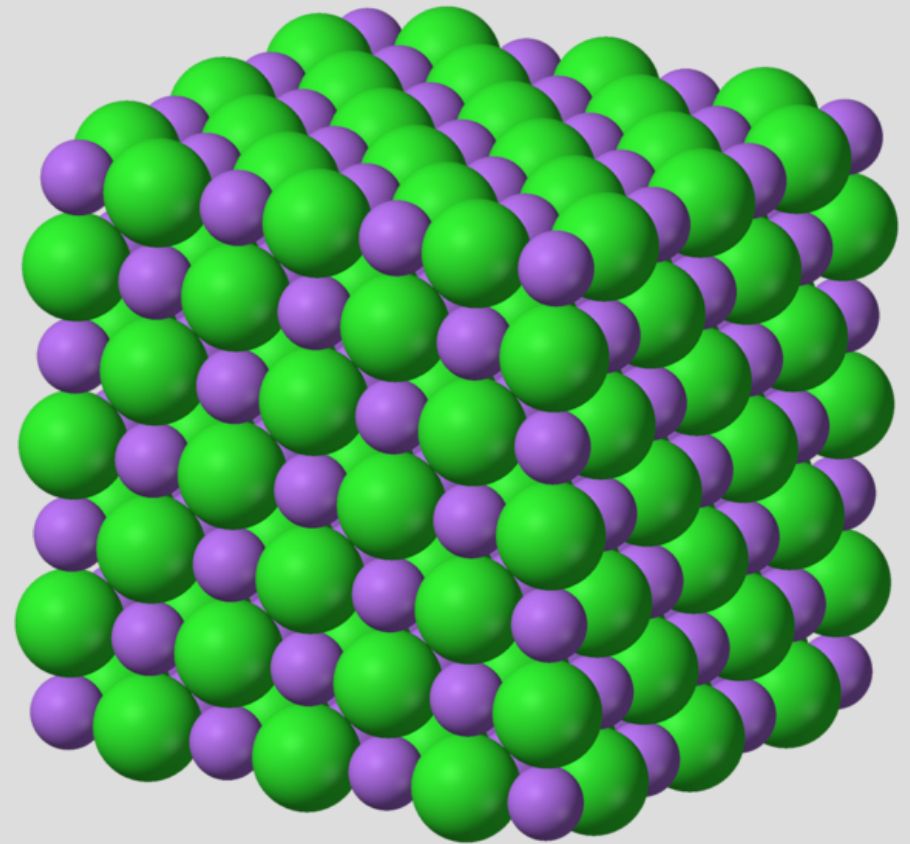
Growth processes

Stephan Matthiesen

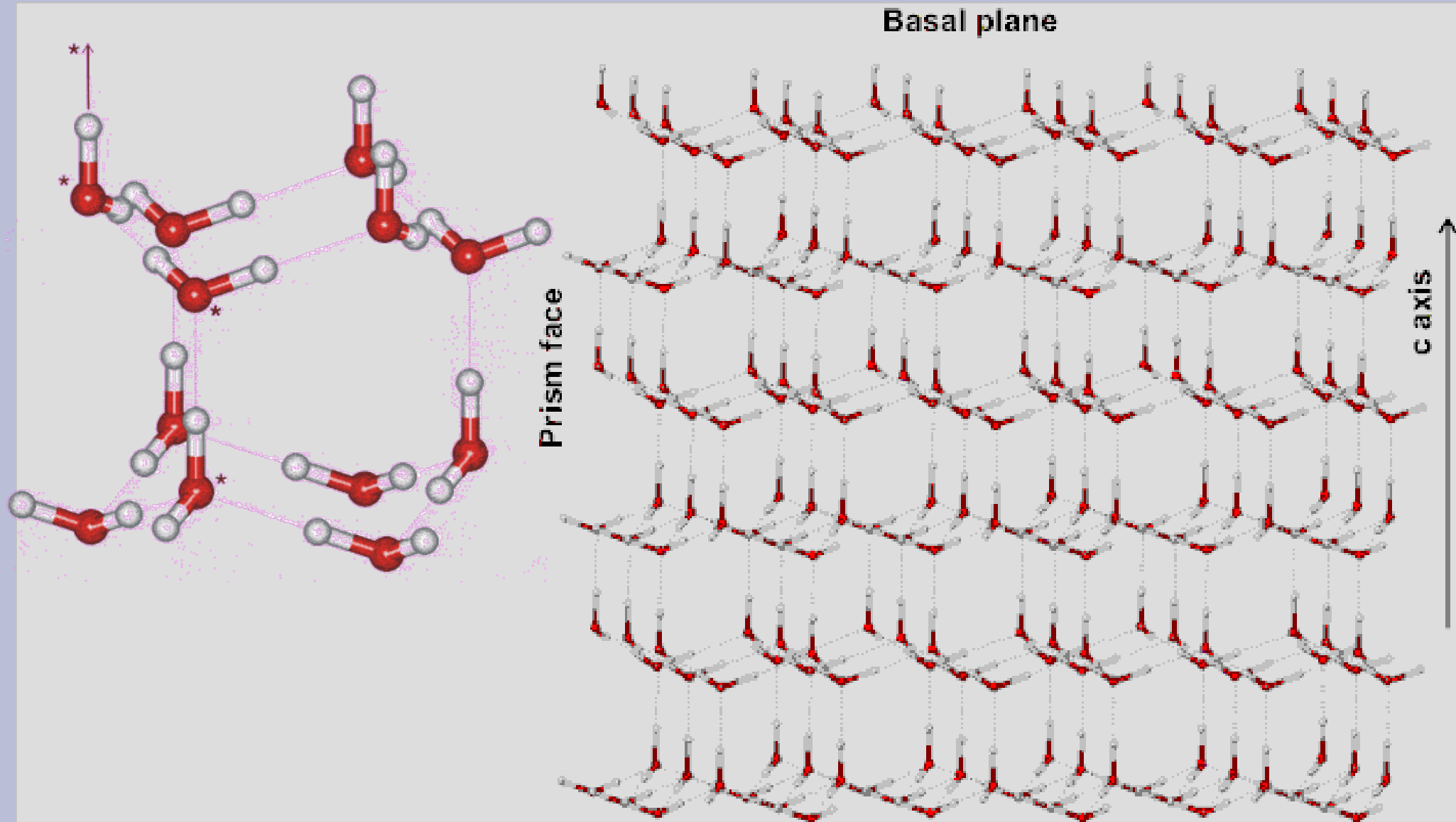
Crystals



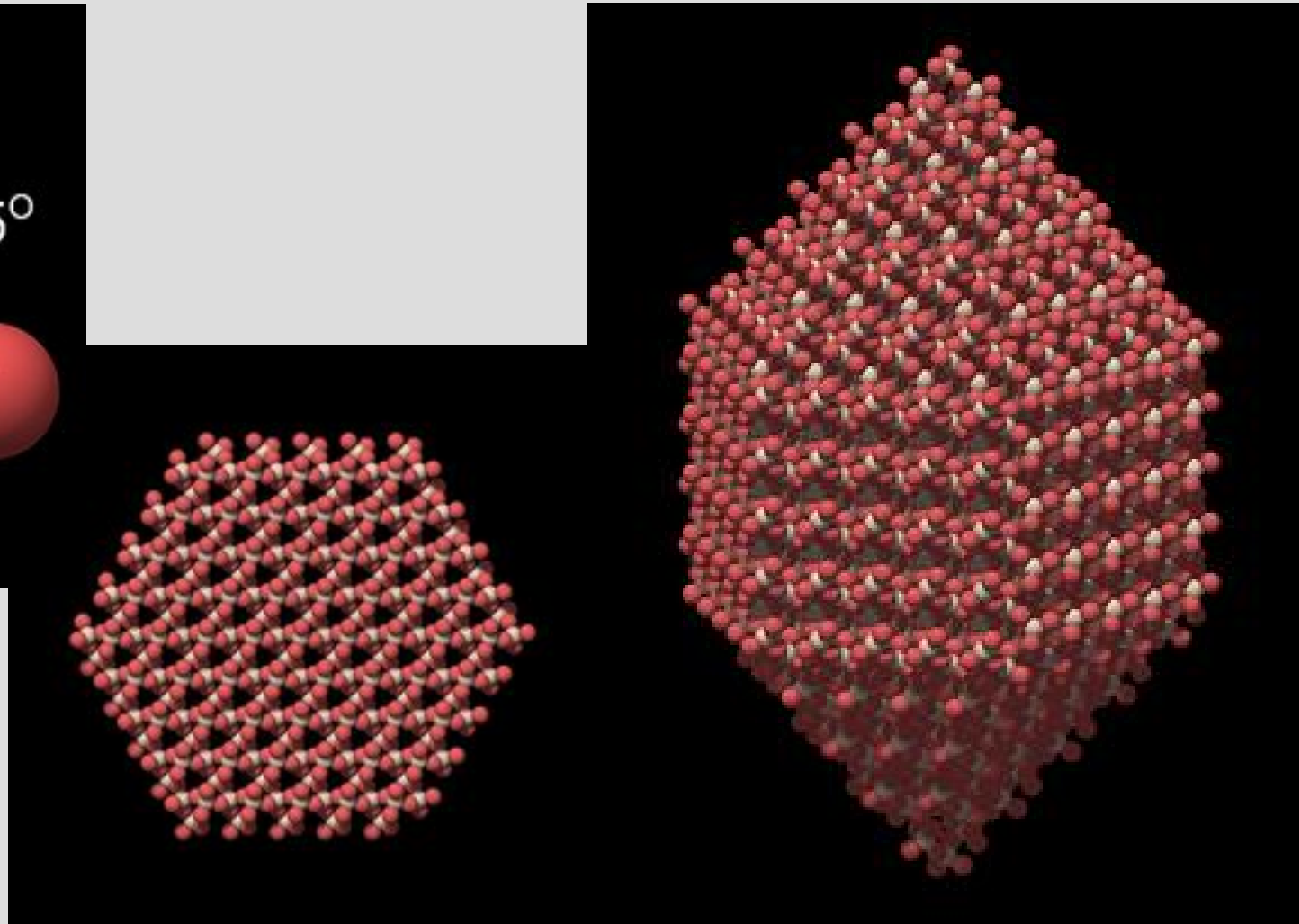
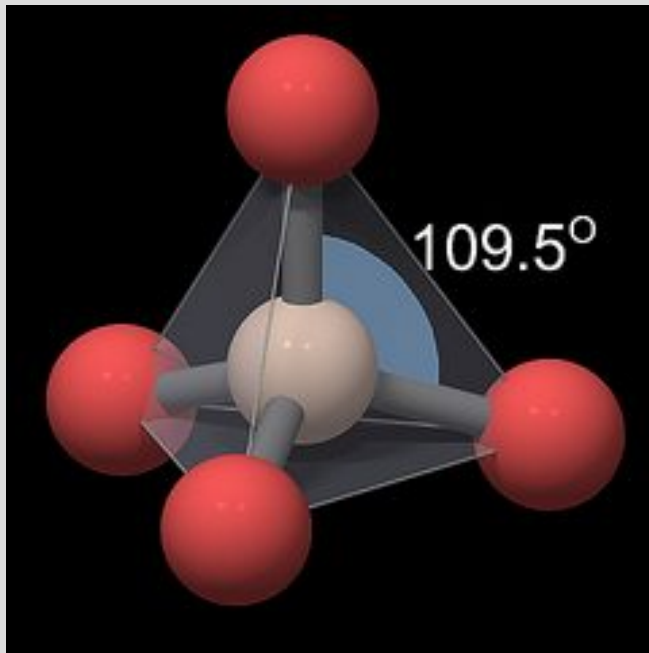
Salt (Sodium Chloride)



Water Ice (hexagonal Ice I_h)

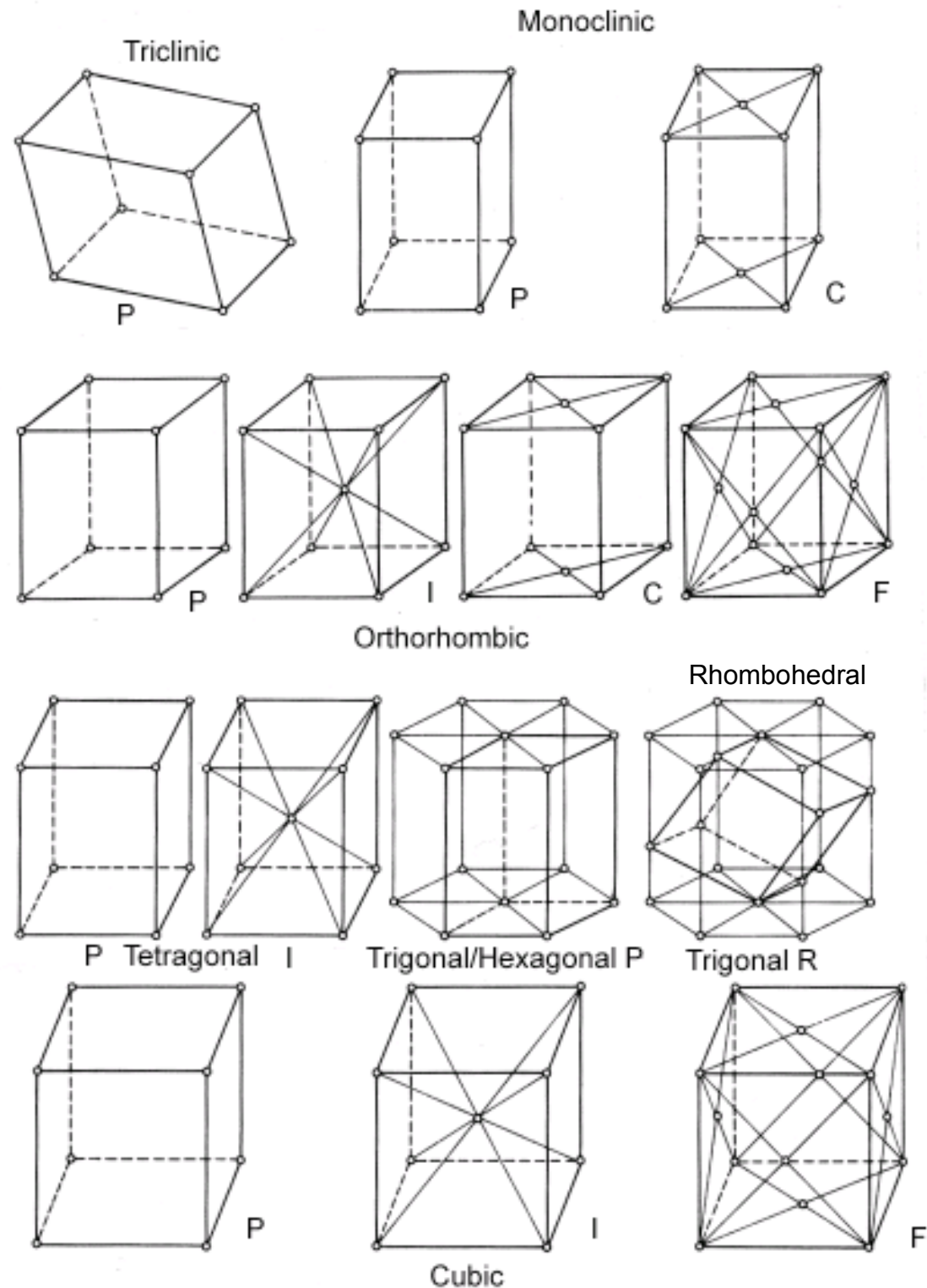


Quartz (SiO_2)

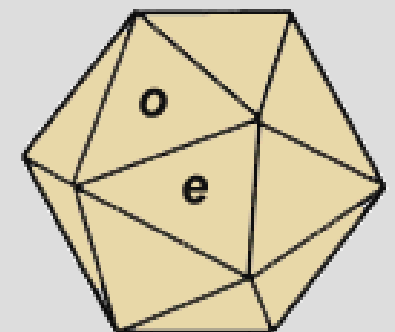
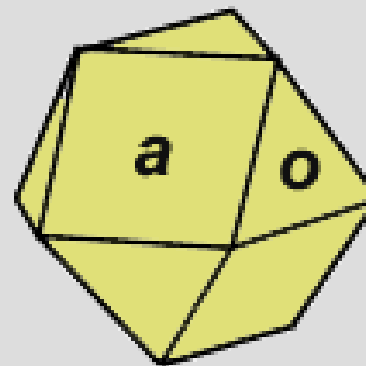
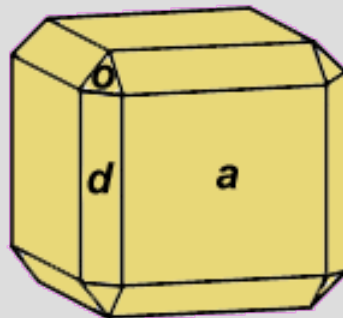
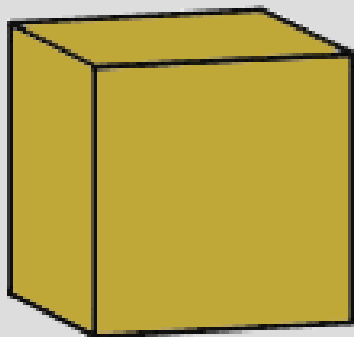
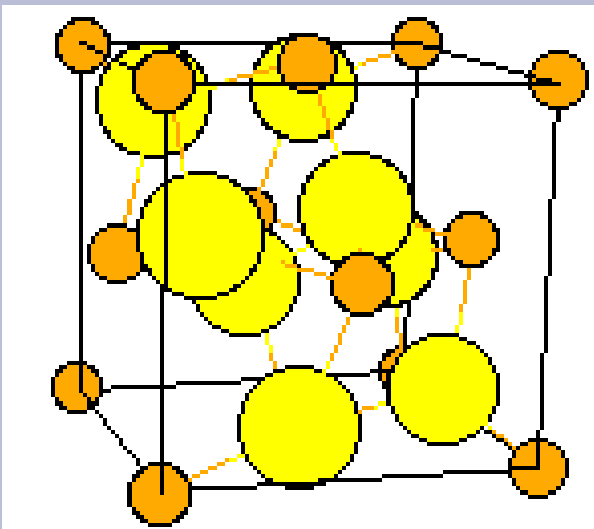
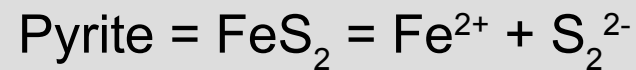


Crystal Systems

- 7 Crystal Systems
- 14 Bravais lattices
- 230 space groups

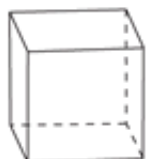


Crystal Forms of Pyrite (cubic)

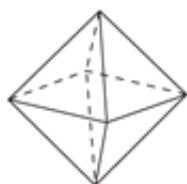


Crystal Forms

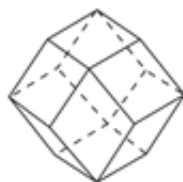
Cubic



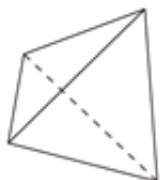
Cube



Octahedron



Dodecahedron



Tetrahedron

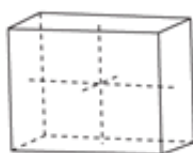


Pyritohedron

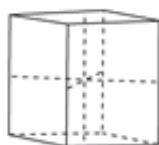


Cube & Pyritohedron

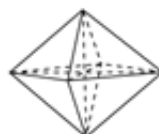
Orthorhombic



Pinacoids



Prism and Basal Pinacoid



Pyramid

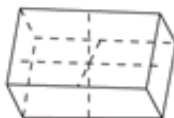


Orthorhombic Sphenoid and Prism



Prism, Domes and Two Pinacoids

Monoclinic



Domes and Pinacoid

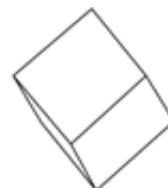


Prism and Pinacoid

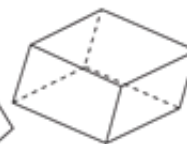
Triclinic



Trigonal



Rhombohedra

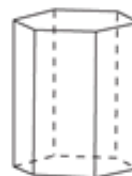


Trigonal Trapezohedron



Trigonal Scalenohedron

Hexagonal



Hexagonal Prism and Base



Hexagonal Pyramid



Hexagonal Prism and Pyramid

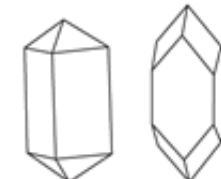
Tetragonal



Tetragonal and Ditetragonal Prism and Base

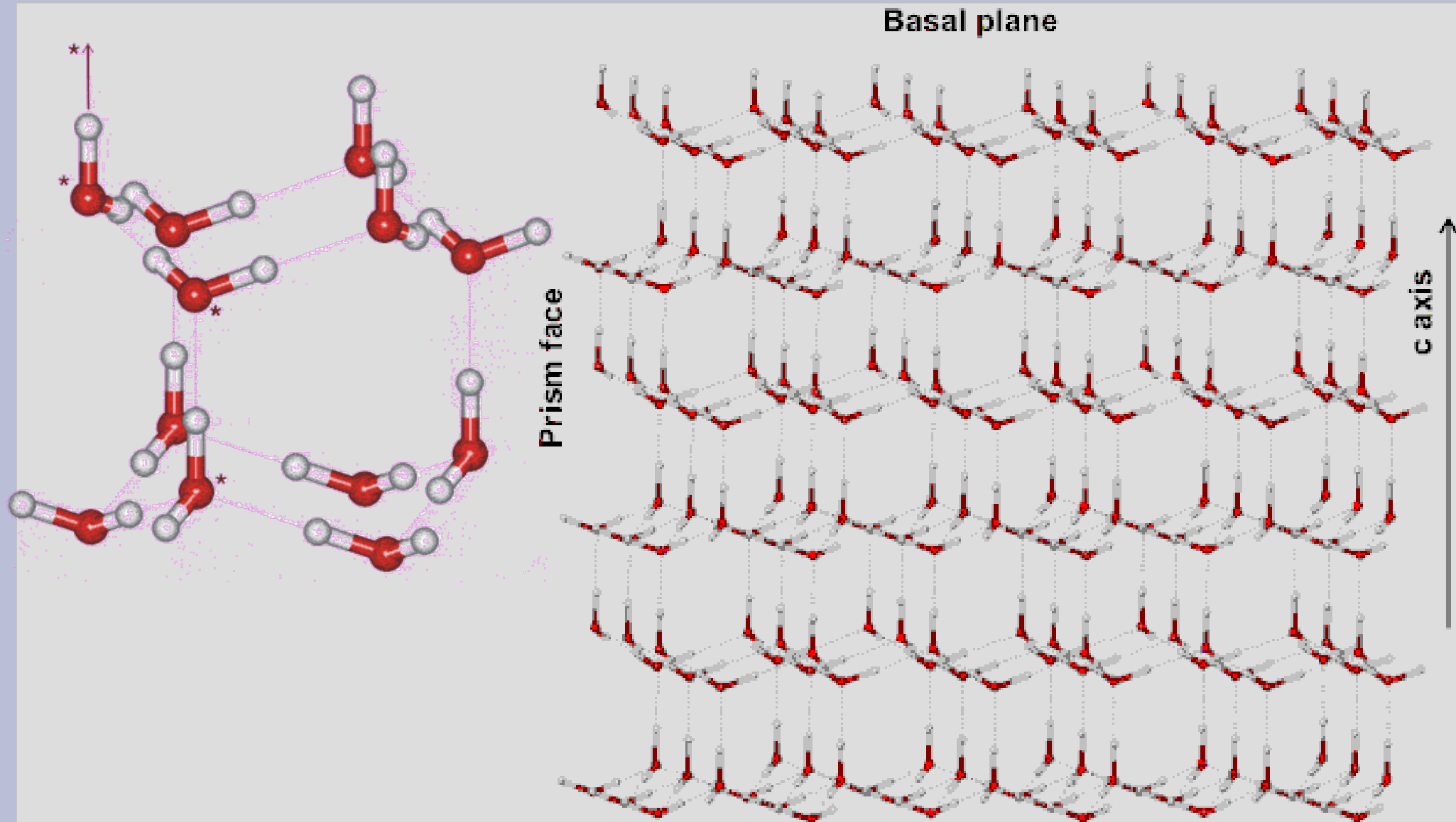


Tetragonal Pyramid

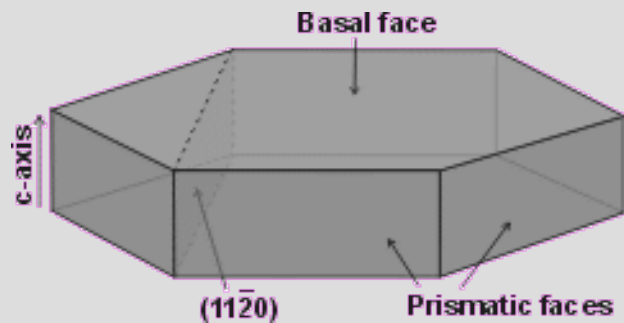
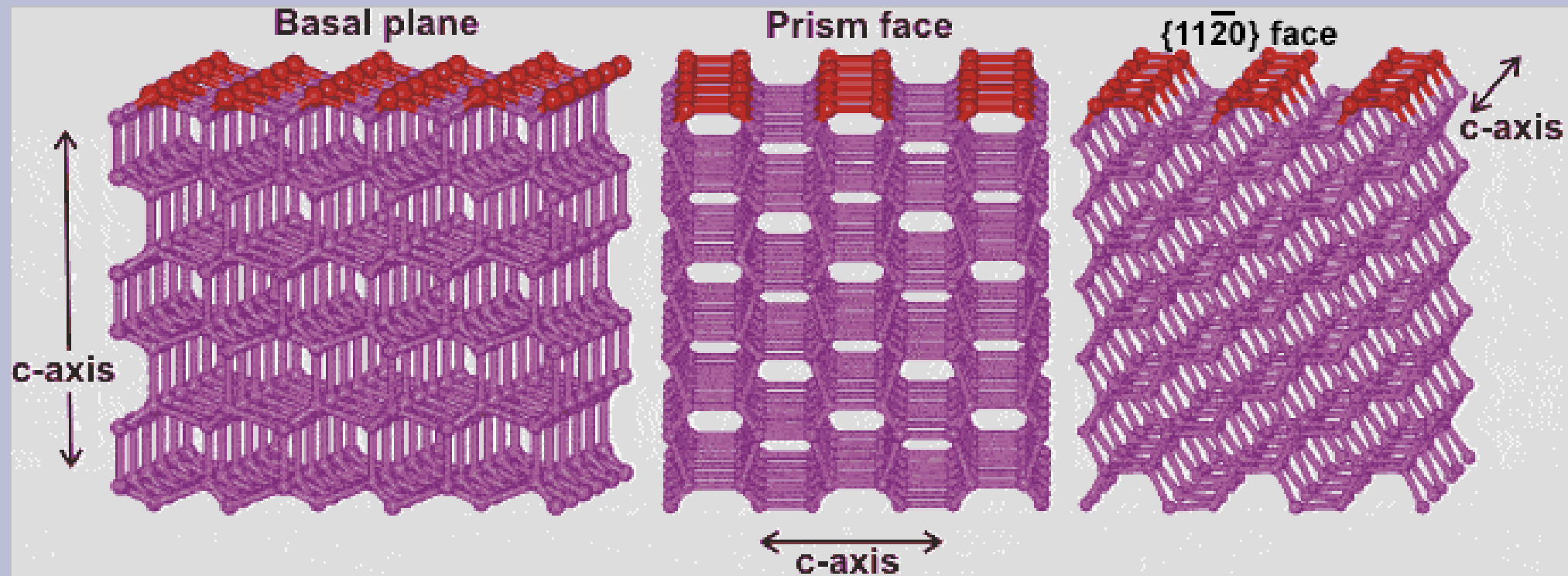


Prism and Pyramid

Water Ice (hexagonal Ice I_h)

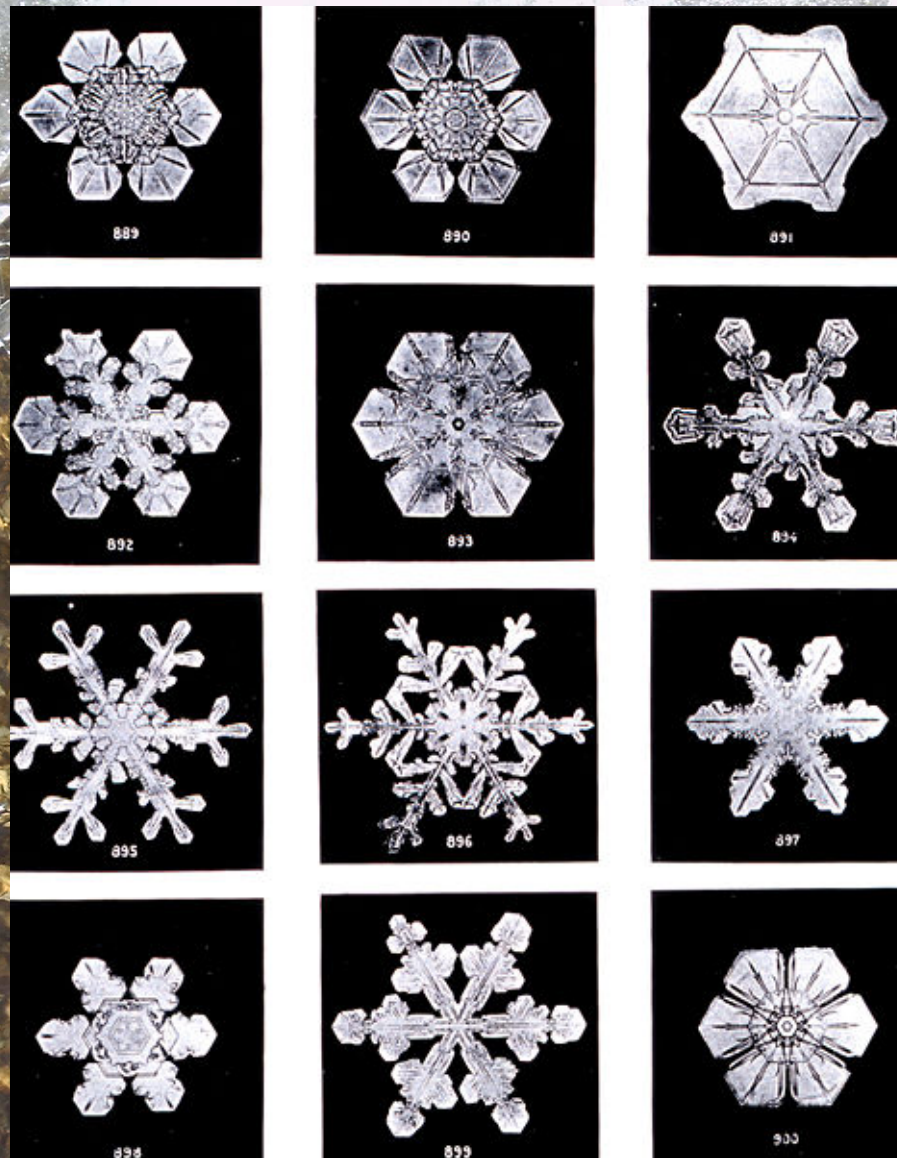


Ice Ih

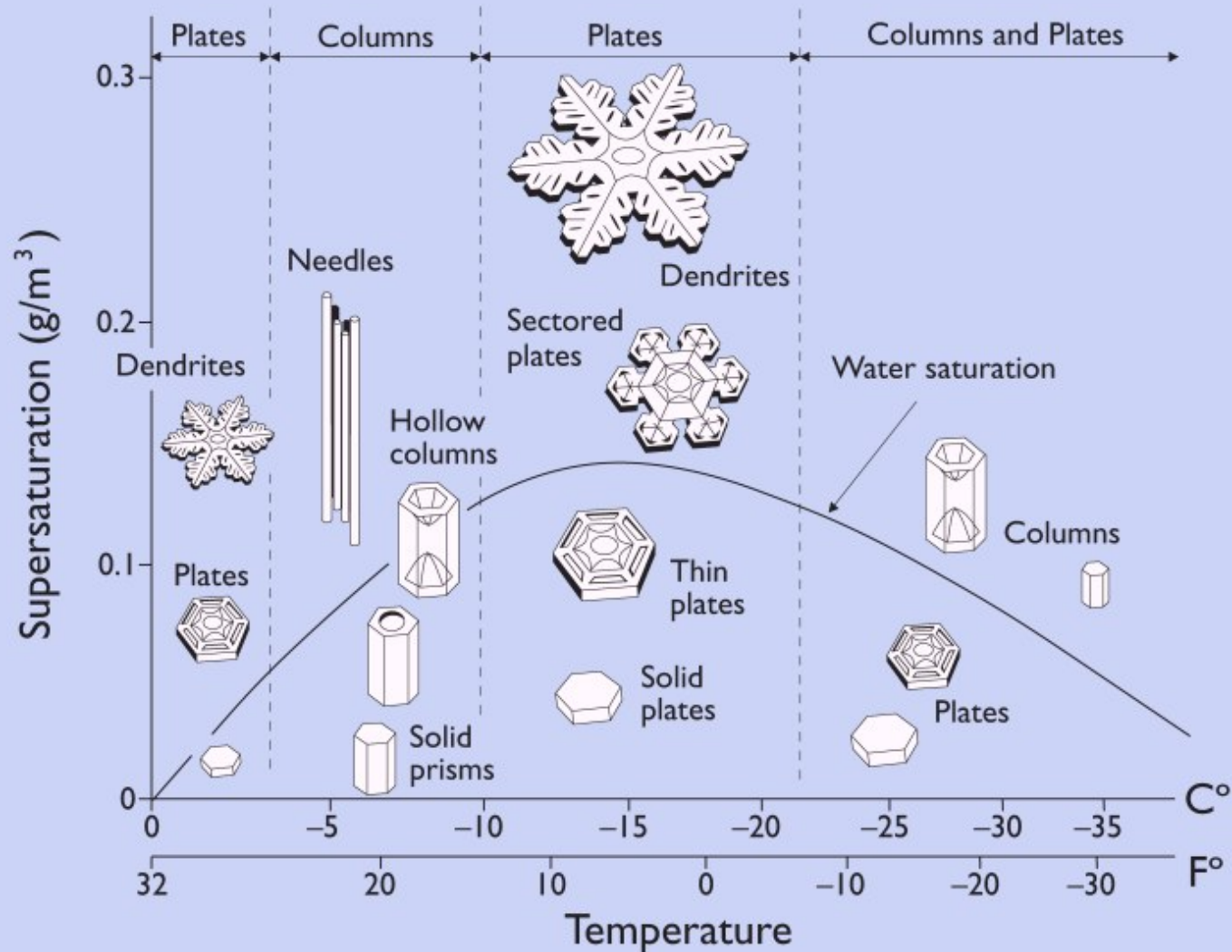


Snowflakes

Wilson Bentley (1865-1931)



Snowflake morphology (Nakaya diagram)



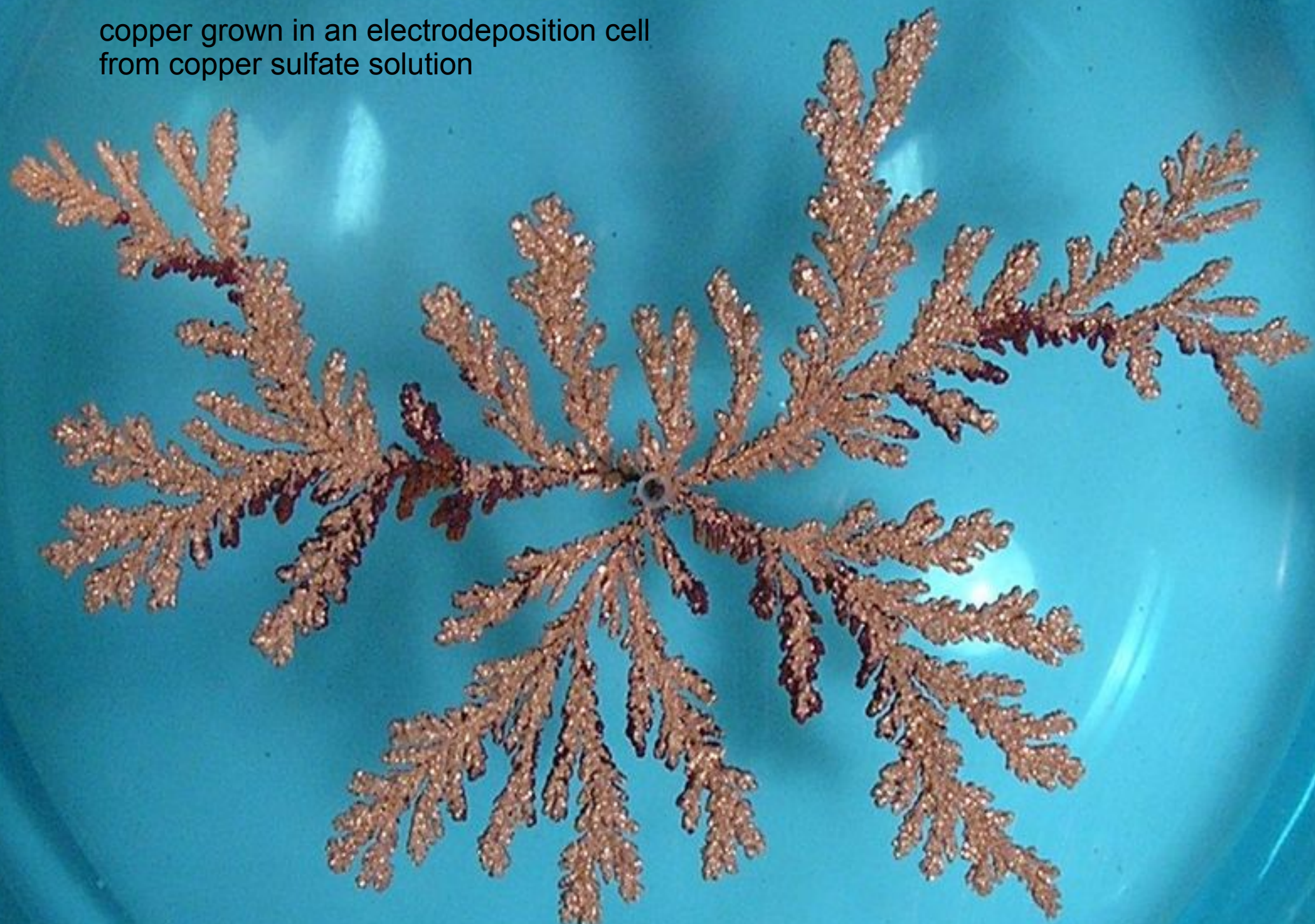
Snowflake morphology depends on growth conditions

- temperature
- water vapour

Diffusion-limited aggregation (DLA)

- particles undergoing **random walk**
(diffusion, Brownian motion)
- aggregate
- no reorganization
- produce “Brownian trees”

copper grown in an electrodeposition cell
from copper sulfate solution



Other fingering mechanisms

- Basic idea: Growth at the tip is easier than at the base
- A lot of different examples and names:
 - Viscuous fingering; flow in porous medium (Saffman-Taylor instability)
 - Fingering in solidification (Mullins-Sekerka instability)









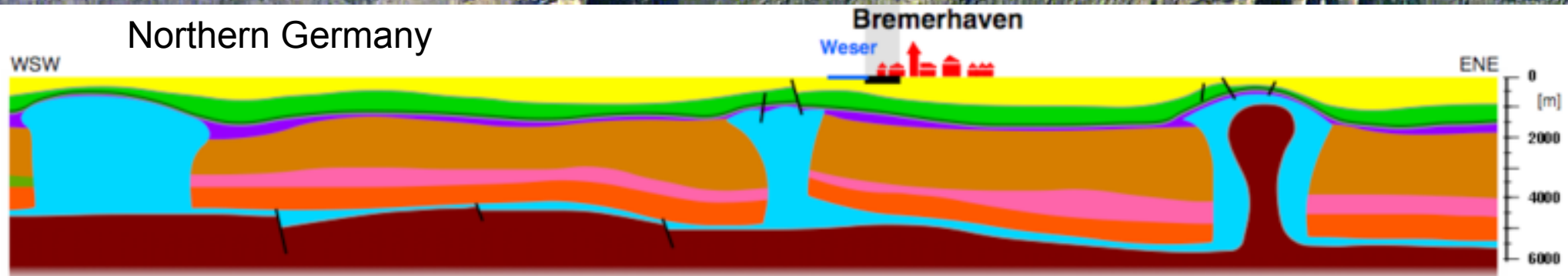
http://en.wikipedia.org/wiki/Salt_dome

Zagros Mountains, Iraq/Iran

Salt domes



Northern Germany



Phyllotaxis



Pineapple science

- How many spiral arms do you count?





34 and 55 spirals



Fibonacci sequence

- 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...
- Each number is the sum of the two predecessors

Leonardo of Pisa (c. 1170 – c. 1250), also known as Leonardo Pisano, Leonardo Bonacci, Leonardo Fibonacci

His father Guglielmo Bonacci directed a trading post in Bugia (now Bejaia, Algeria), he later lived in Sicily and Pisa.

Fibonacci introduced the Hindu-Arabic number system to Europe in his *Liber Abaci* (Book of Abacus calculations)



Fibonacci sequence and Golden Ratio

Fibonacci: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

$$2 / 1 = 2$$

$$3 / 2 = 1.5$$

$$5 / 3 = 1.66666...$$

$$8 / 5 = 1.6$$

$$13 / 8 = 1.625$$

$$21 / 13 = 1.61538461538...$$

$$34 / 21 = 1.61904761905...$$

$$55 / 34 = 1.61764705882...$$

$$89 / 55 = 1.61818181818...$$

$$\lim(f_n / f_{n-1}) = 1.61803398... = (1 + \sqrt{5}) / 2$$

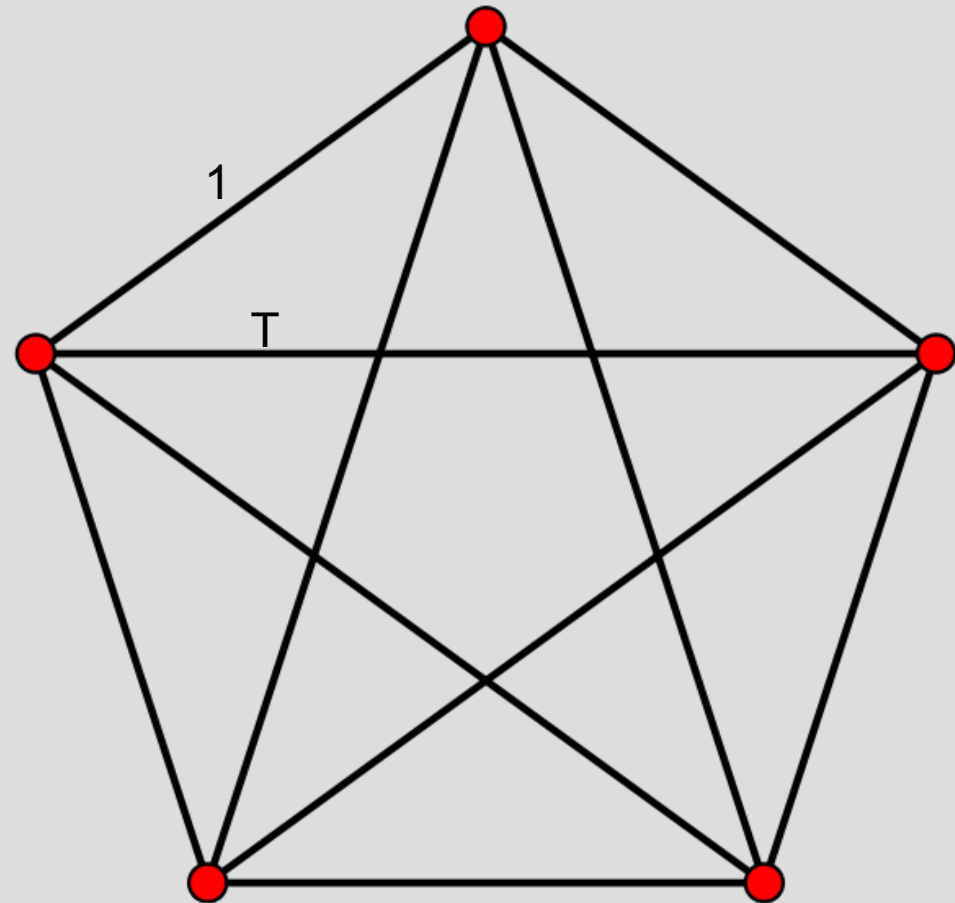
Golden Ratio

$$T = \lim(f_n/f_{n-1}) = 1.61803398... = (1 + \sqrt{5})/2$$

$$T : 1 = (T + 1) : T$$

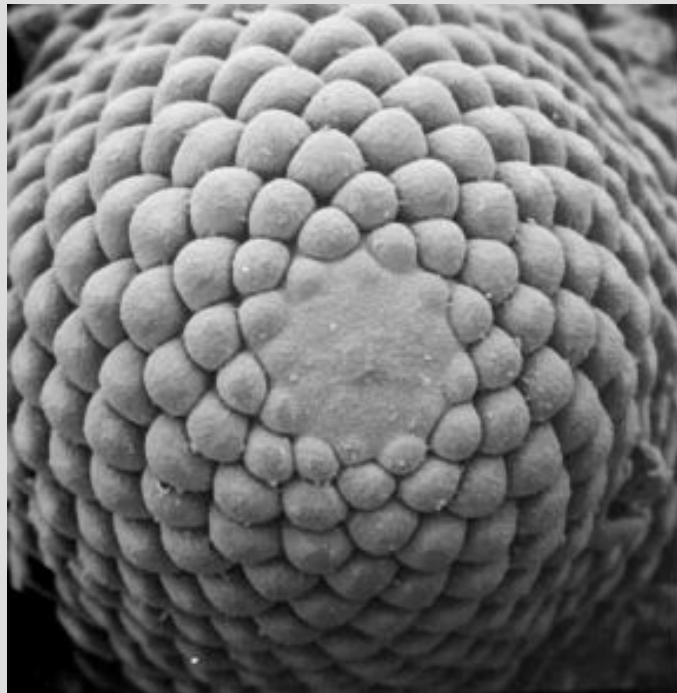
Golden angle:
 $360^\circ/T \sim 222.5...^\circ$

Or, more common,
make it smaller than 180° :
 $360^\circ - 222.5...^\circ \sim 137.5...^\circ$



Phyllotaxis or phyllotaxy (arrangement of leaves)

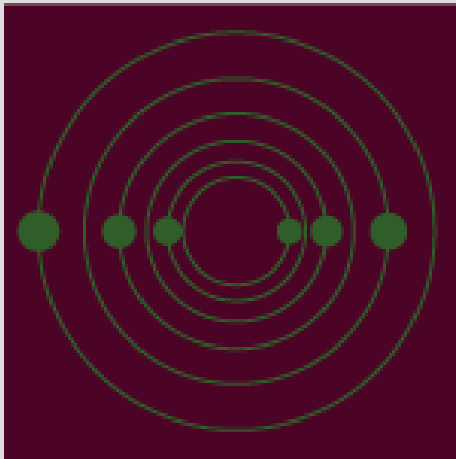
- plants: modular organisms
- tip (apex) growth:
new modules formed on
meristematic ring



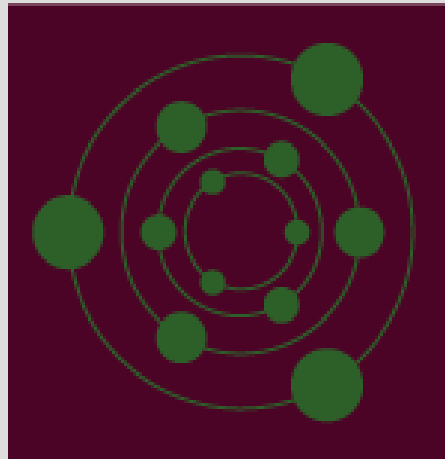
Classification:

Main types of phyllotaxis

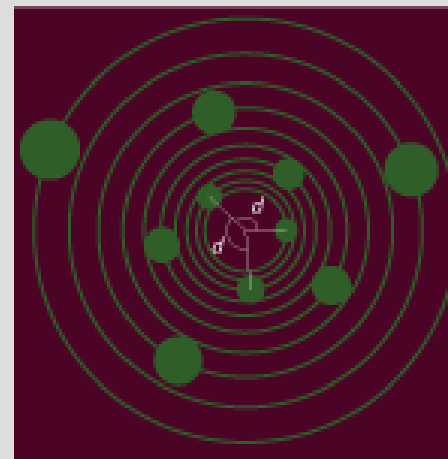
distichous



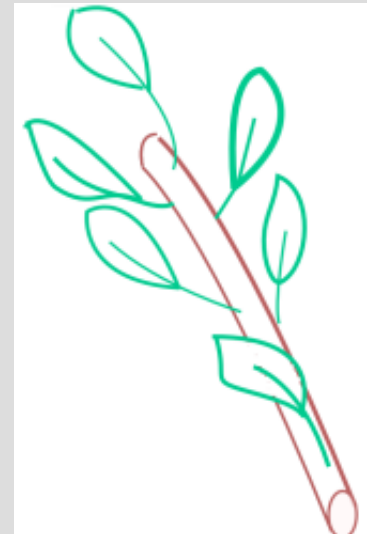
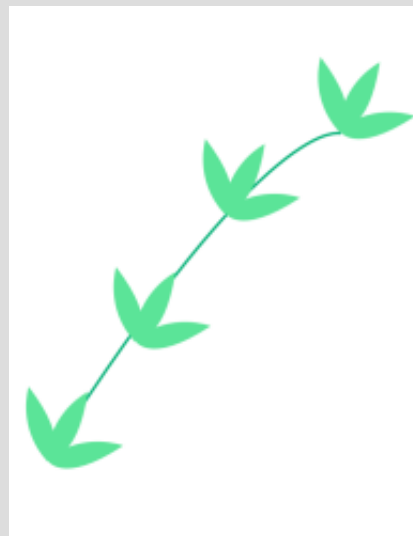
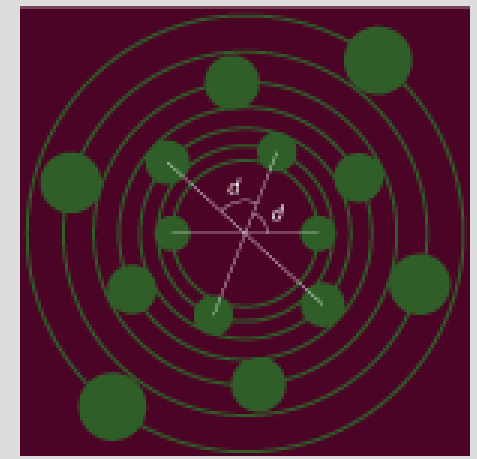
whorled



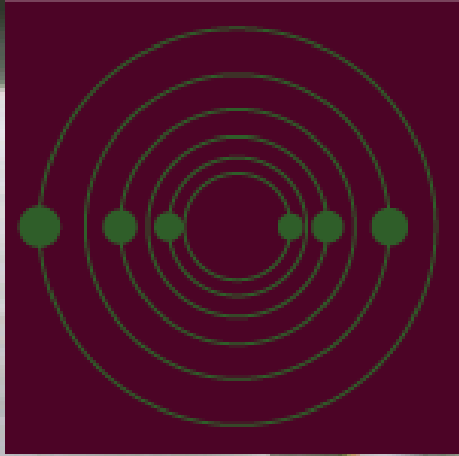
spiral



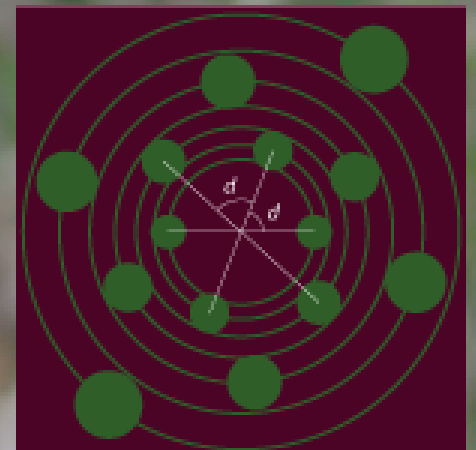
multijuguous



distichous



multijuguous

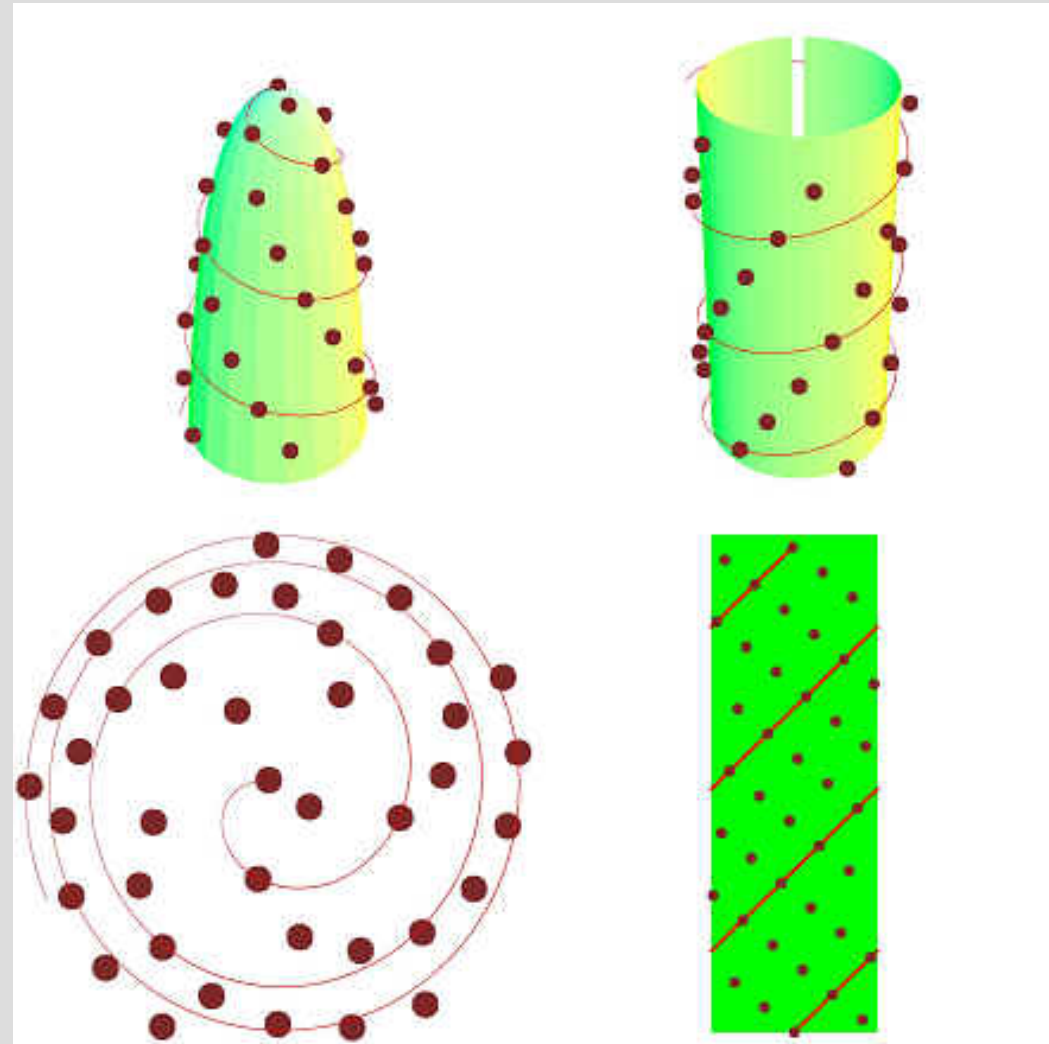
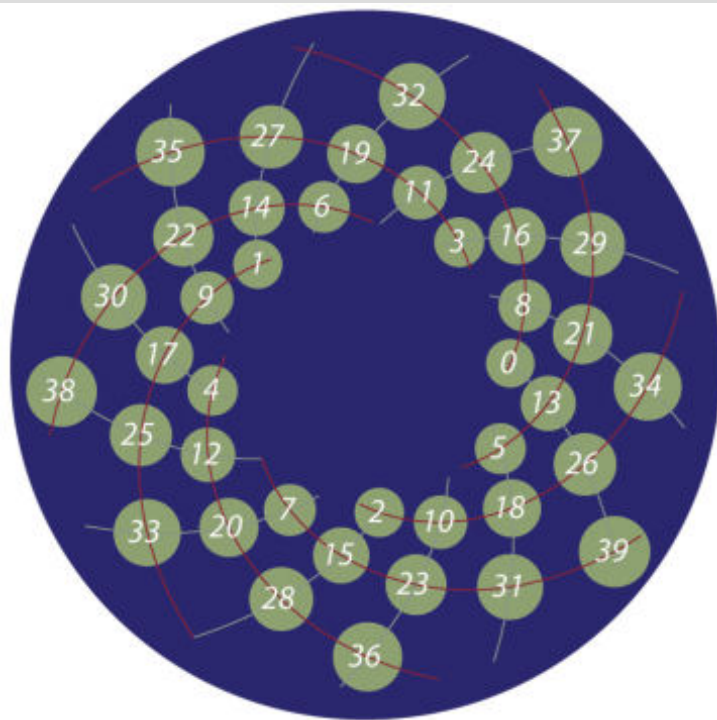




Spiral

Two spirals...

- paristiche: spirals made up of next neighbours
- **generative spiral**

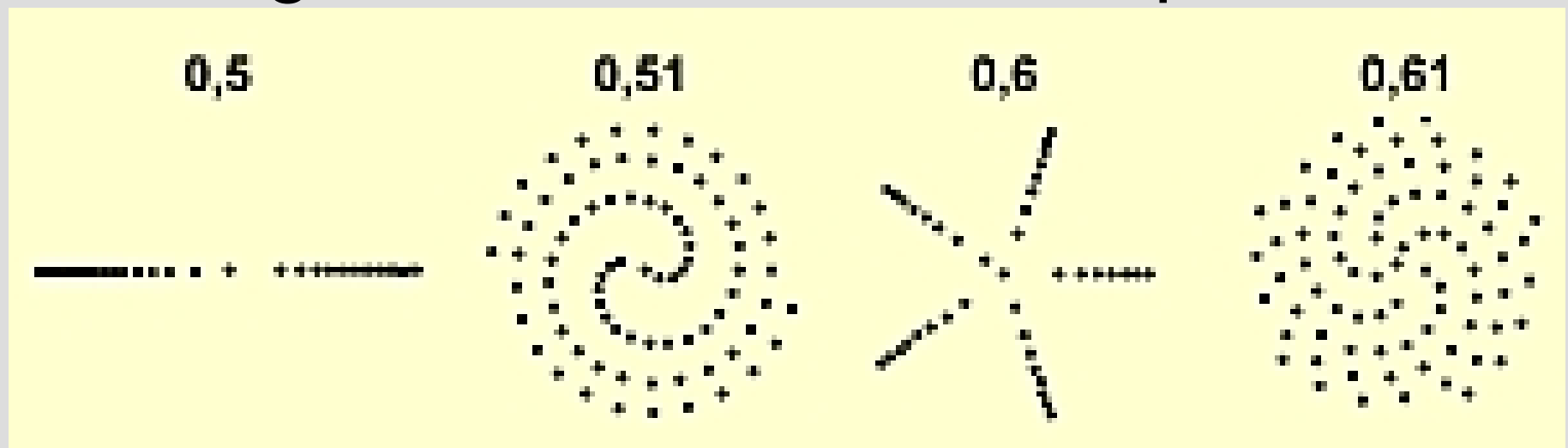


Why the golden angle?

Try different angles with the
Spiral lattices applet:

<http://www.math.smith.edu/phylo//Applets/Spiral/Spiral.html>

Golden angle gives the densest
arrangement with least overlap



The most irrational number

There is a „most irrational“ number, and it turns out (surprise, surprise) the golden number.

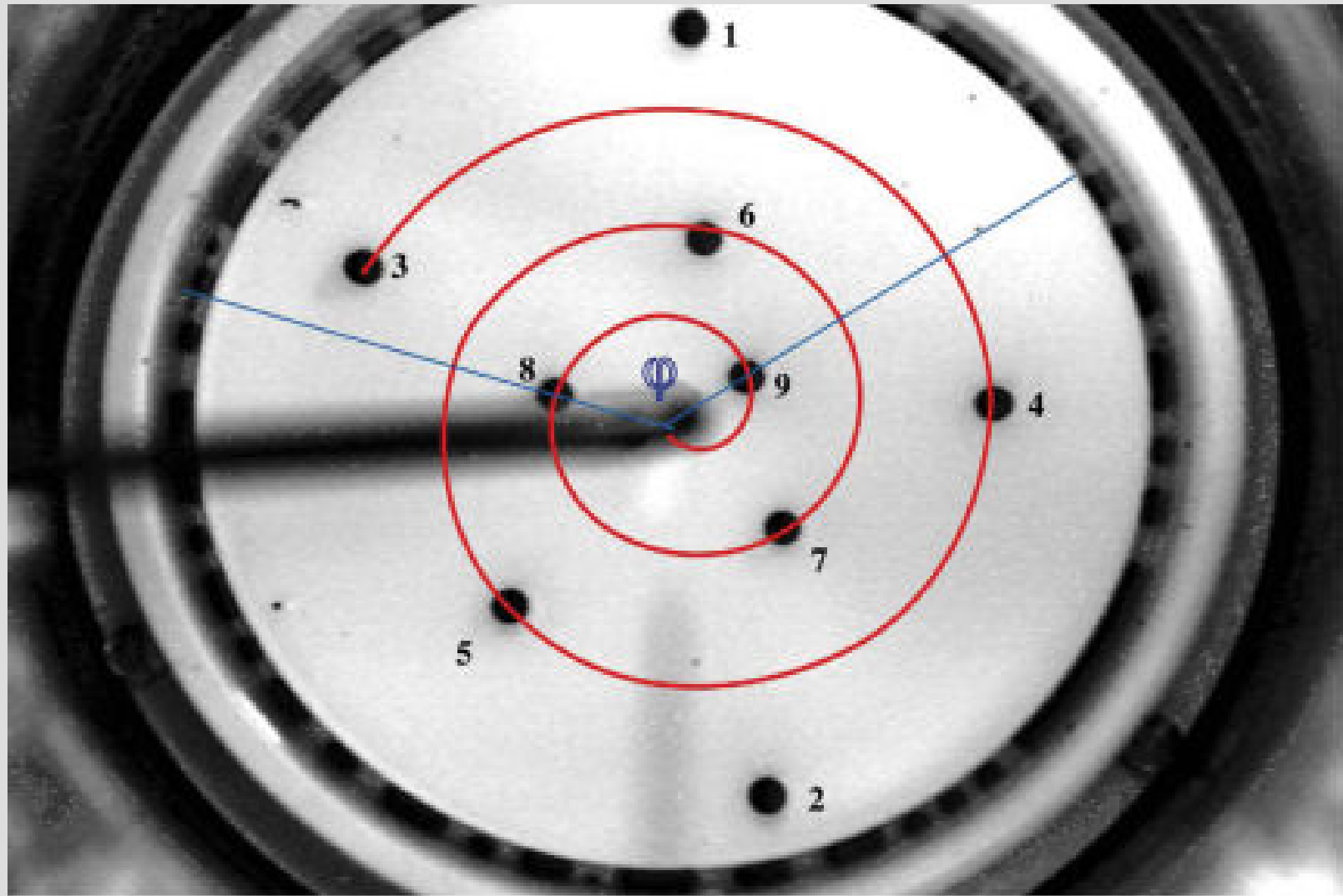
...

It is the most badly „approximable-by-rational“ number there is!

Its „badness“ is exceeded only by the awkwardness of the preceding sentence.

(Adam, Mathematics in Nature, p220)

An Experiment: Phyllotaxis as self-organization



Patterns in Nature Outline

1. Introduction
2. Waves and oscillations
3. Regularity and chaos
4. Animal cooperation
5. Spatial patterns
6. Aggregation and growth processes
7. Cellular automata
8. Fractals
9. Miscellaneous topics
10. Concluding session

