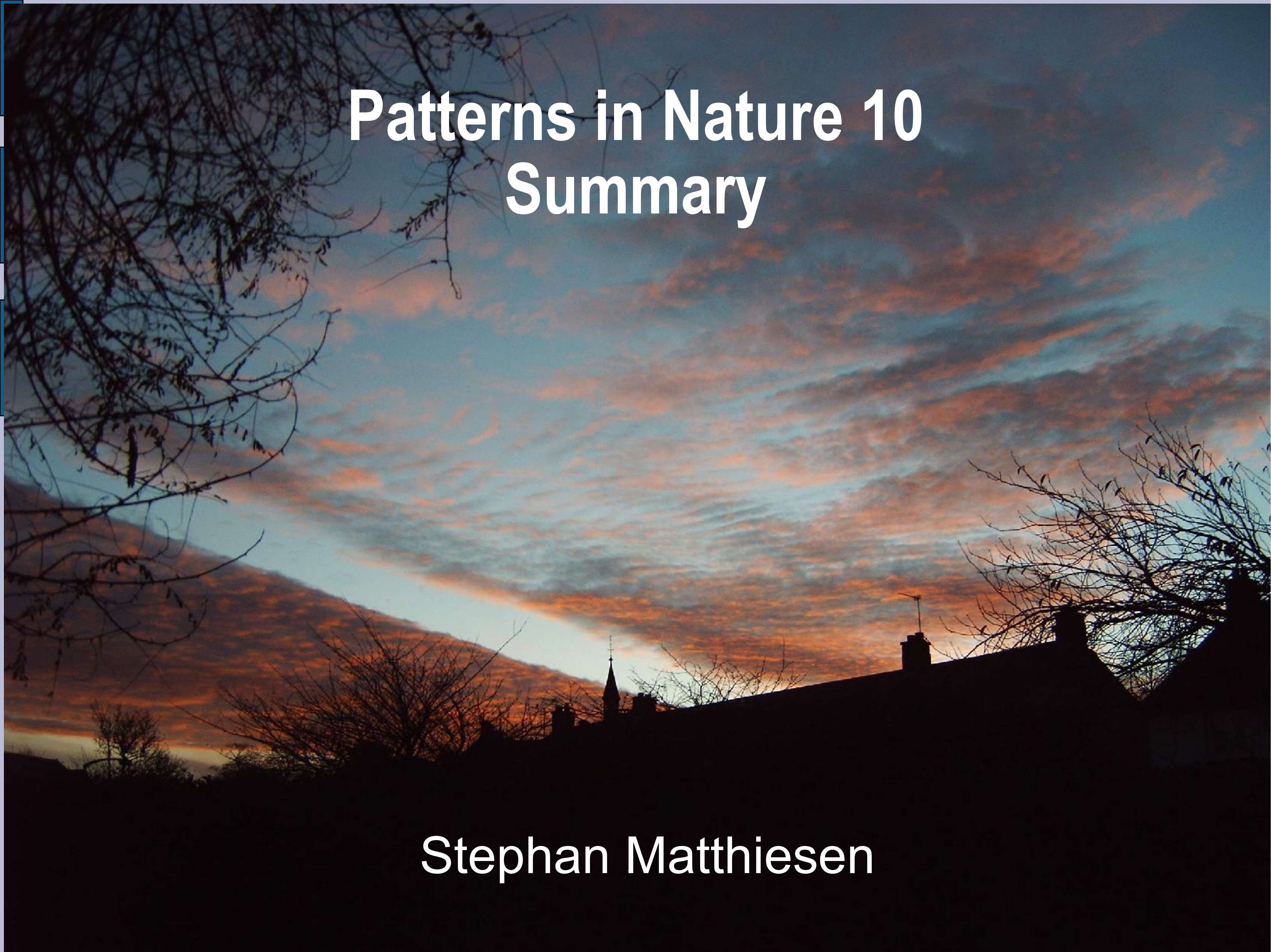


Patterns in Nature 10
Summary

Stephan Matthiesen

Pattern formation in the human world

- Well-informed leader
- Building by blueprint
- Following a recipe
- Templates

Usually, several of these mechanisms interact.

The instructions are “external”.

Stygmergy

Modifying the process through feedback from the emerging pattern.

Examples:

- A cook tasting the dish and modifying the recipe.
- Tourists going into the pub where there is already a crowd.

Self-organization Definition

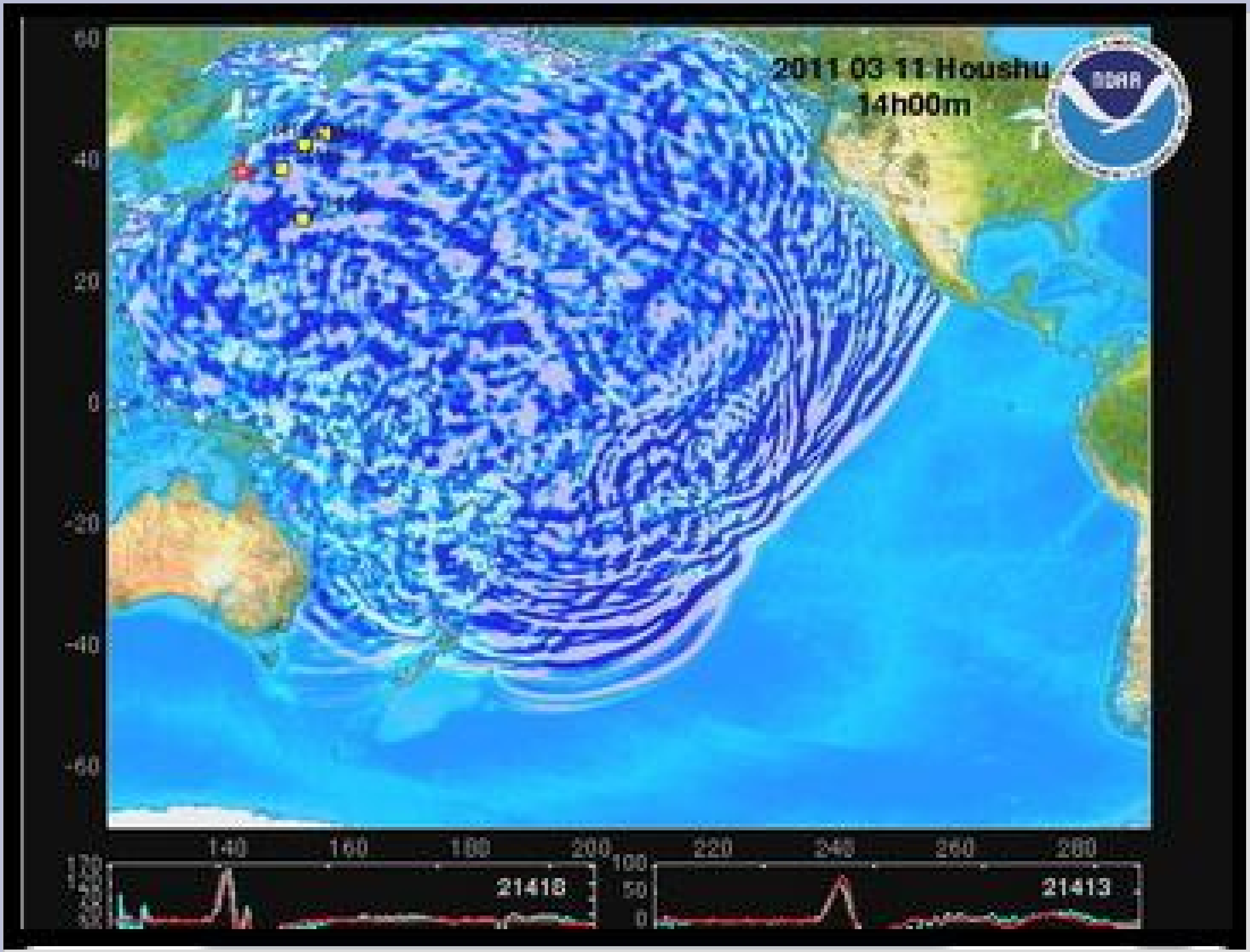
Self-organization is a process in which pattern at the global level of a system emerges solely from numerous interactions among the lower-level components of the system. Moreover, the rules specifying interactions among the system's components are executed using only local information, without reference to the global pattern.

(Camazine et al 2001, p. 8)

Characteristics of waves

- **Superposition** – when waves meet, the total displacement is the sum of the displacements
- **Interference** – superposition of waves to form a new wave pattern
- **Diffraction** – (circular) spreading from entering a hole of comparable size to their wavelengths
- **Reflection** – direction change from hitting a reflective surface
- **Refraction** – direction change from entering a new medium (with different wave speed)
- **Dispersion** – splitting up by frequency

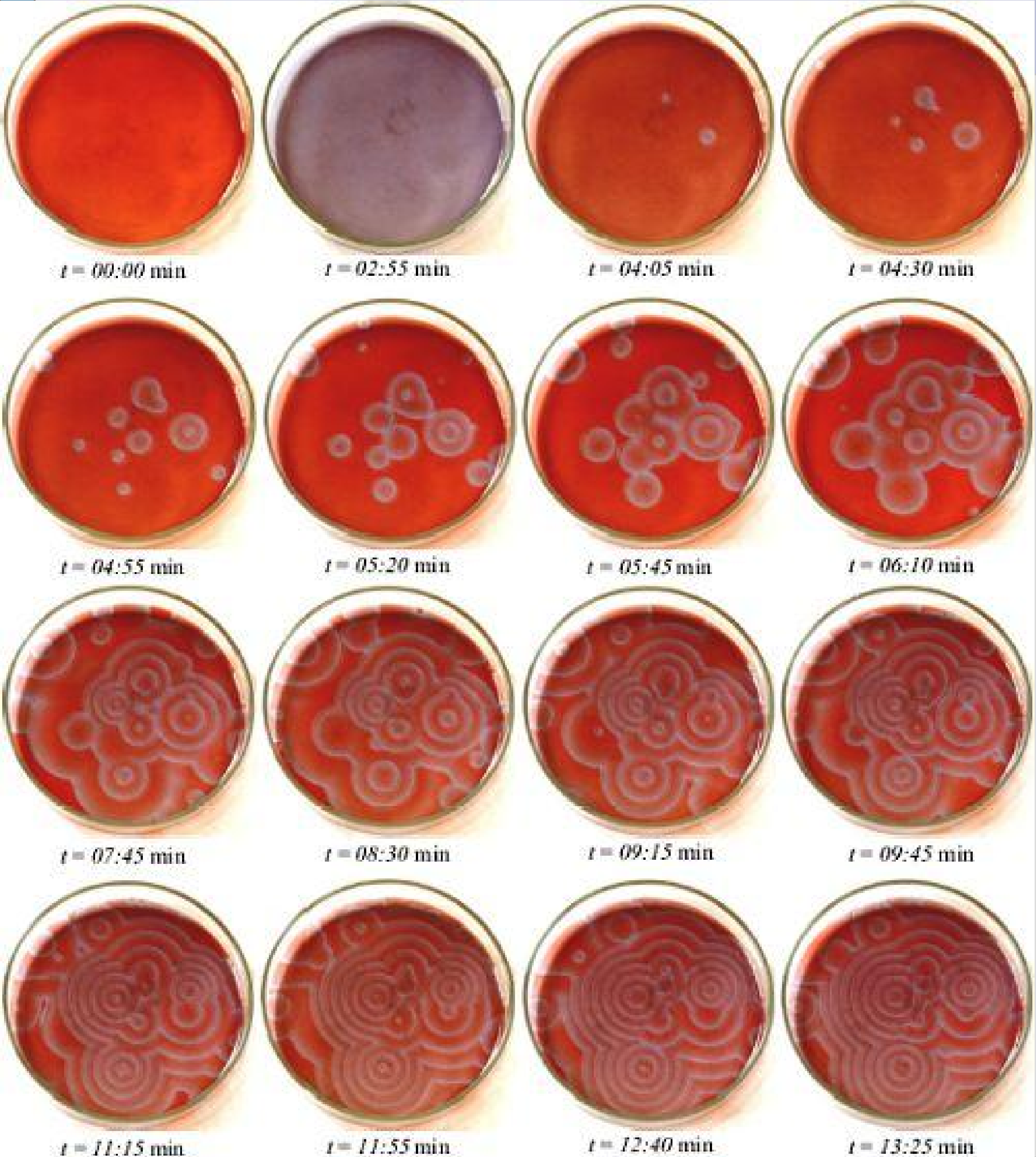
Tsunamis



Honshu event, 11 March 2011, Source: NOAA (http://nctr.pmel.noaa.gov/honshu20110311/)

- wave length: 100-500 km
- period: 1 hour
- wave speed up to 800 km/h (500 mph)

Spatial patterns from BZ



Video: http://uk.youtube.com/watch?v=bH6bRt4XJcw

http://jkrieger.de/bzr/inhalt.html)

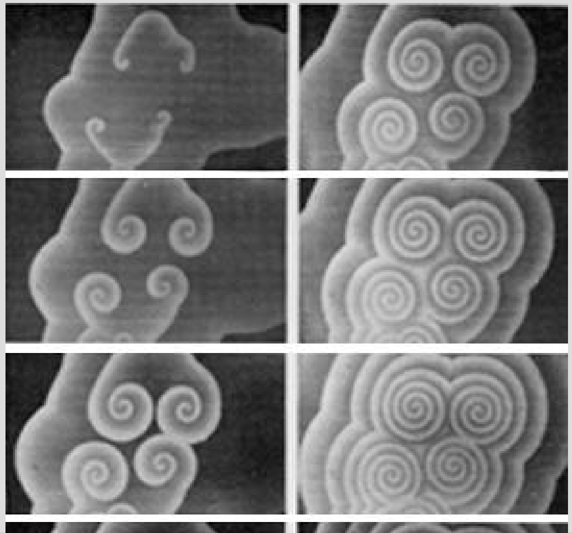
Two types of waves

„Normal“ waves

- Mechanism: restoring force
- Circular shapes
- Interaction
  - Superposition
  - Diffraction

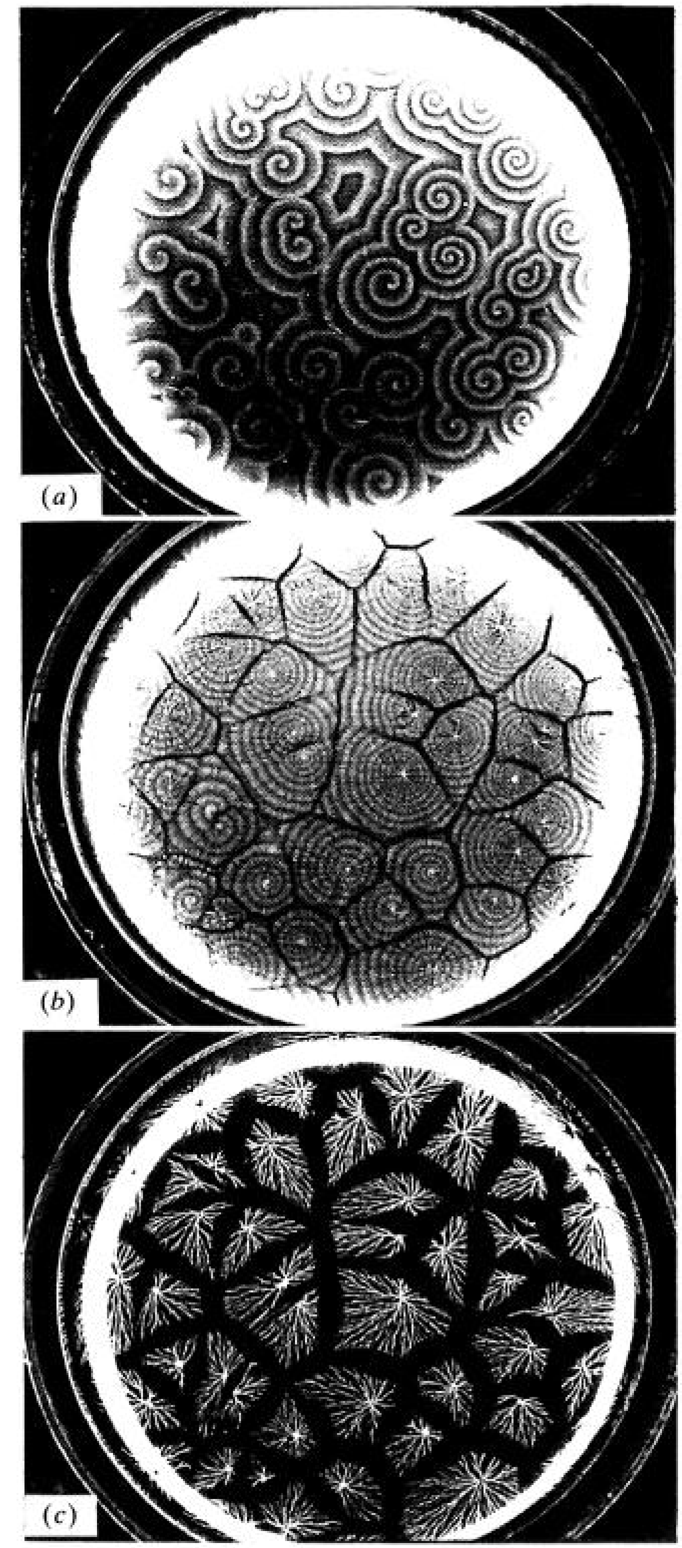
Excitation waves

- Mechanism: excitation/latency
- Spiral shapes
- Interaction
  - Extinction



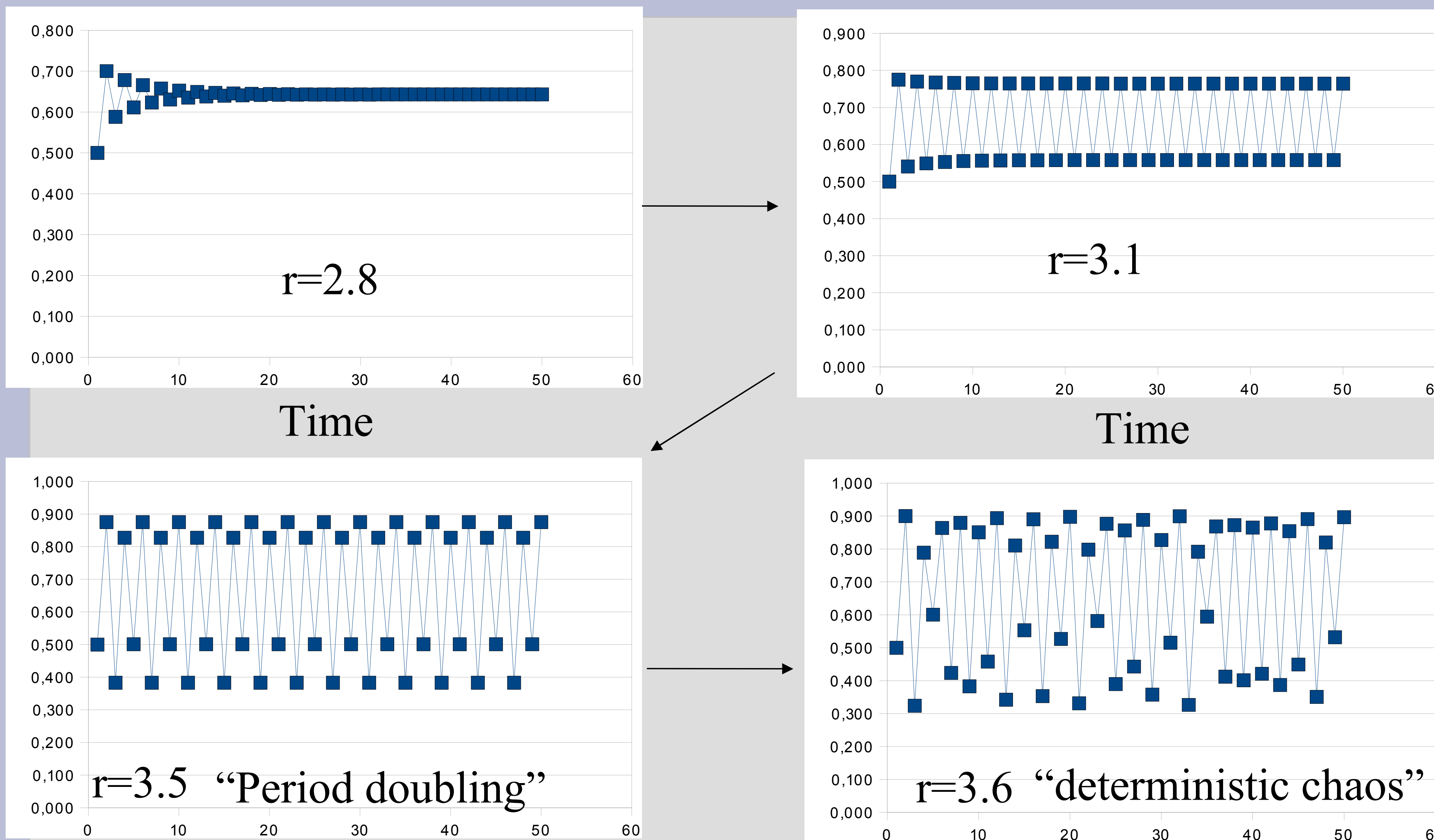
Excitation waves in slime mold

- cAMP: cyclic adenosine monophosphate
- Spiral waves of cAMP induce:
  - (a) Cell movement
  - (b) onset of cell streaming
  - (c) development of stream morphology



Höfer 1995

The logistic map (modelled in a spreadsheet)

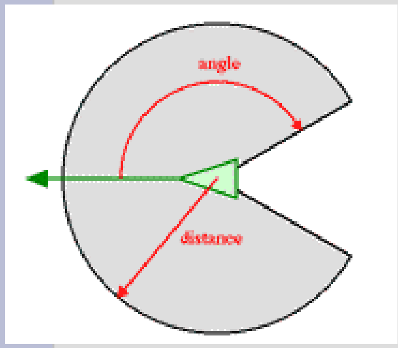


Deterministic Chaos

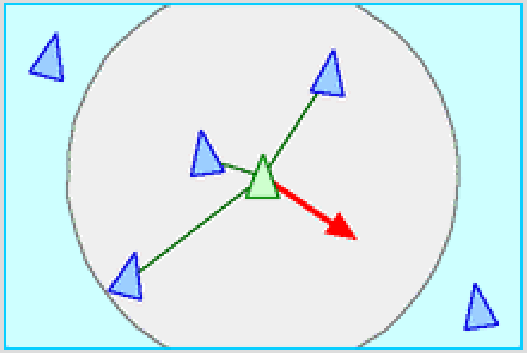
- A system that follows deterministic rules (and therefore is – in principle – completely predictable)
- But it is very sensitive to the initial conditions: small changes alter the result completely (Nonlinearity)
- Therefore, in practice it is not predictable because one can never know all initial conditions with infinite precision.

Understanding swarm behaviour

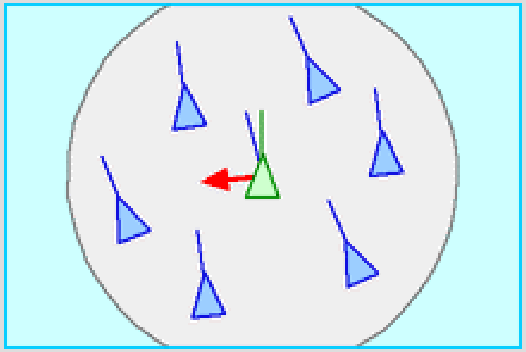
- individual-based model developed by Craig Reynolds (1986)
- "Boids" (elementary "animals"):
  - they react only to their local neighbourhood
  - neighbourhood characterised by distance & angle
  - they follow 3 simple behaviour rules



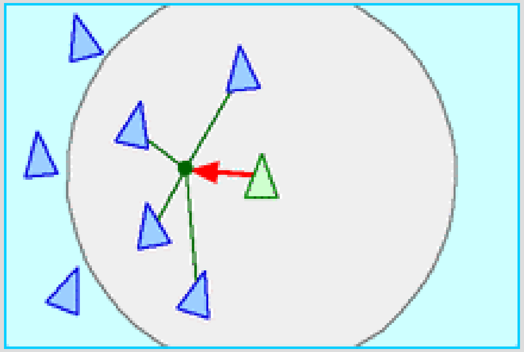
A Boid's neighbourhood



Separation
steer to avoid crowding local flockmates



Alignment
steer towards the average heading of local flockmates



Cohesion
steer to move toward the average position of local flockmates

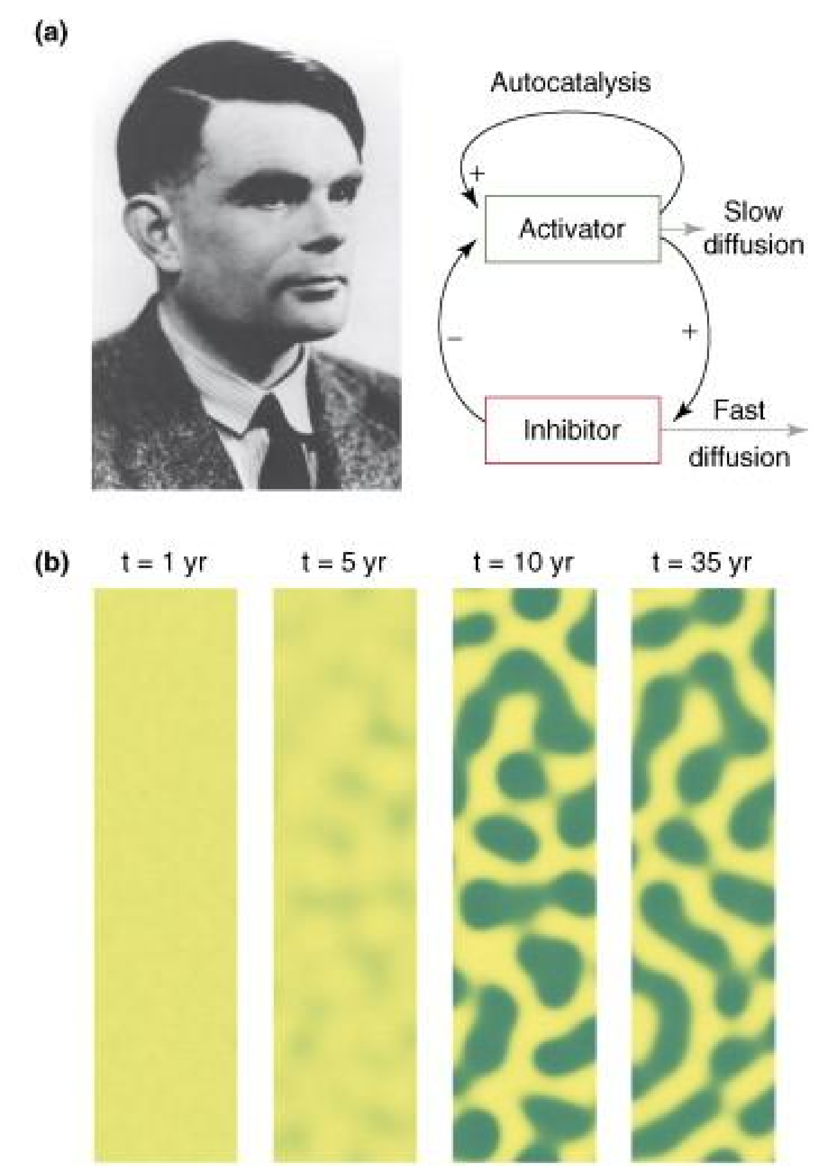
Mass panic in Mecca

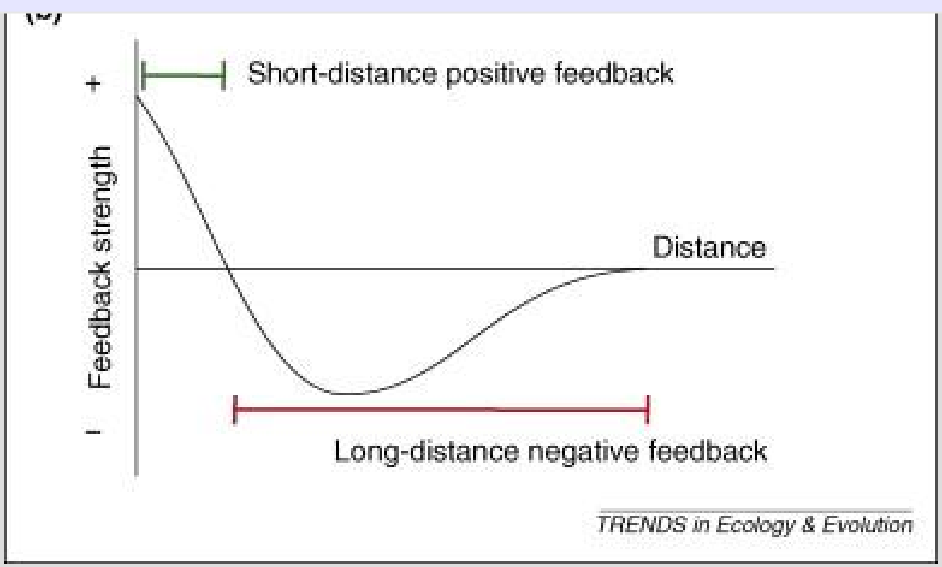
Activator-Inhibitor Model

feedback: part of a system's output influences the input

• positive feedback: the system responds to perturbations in the same direction as the perturbation

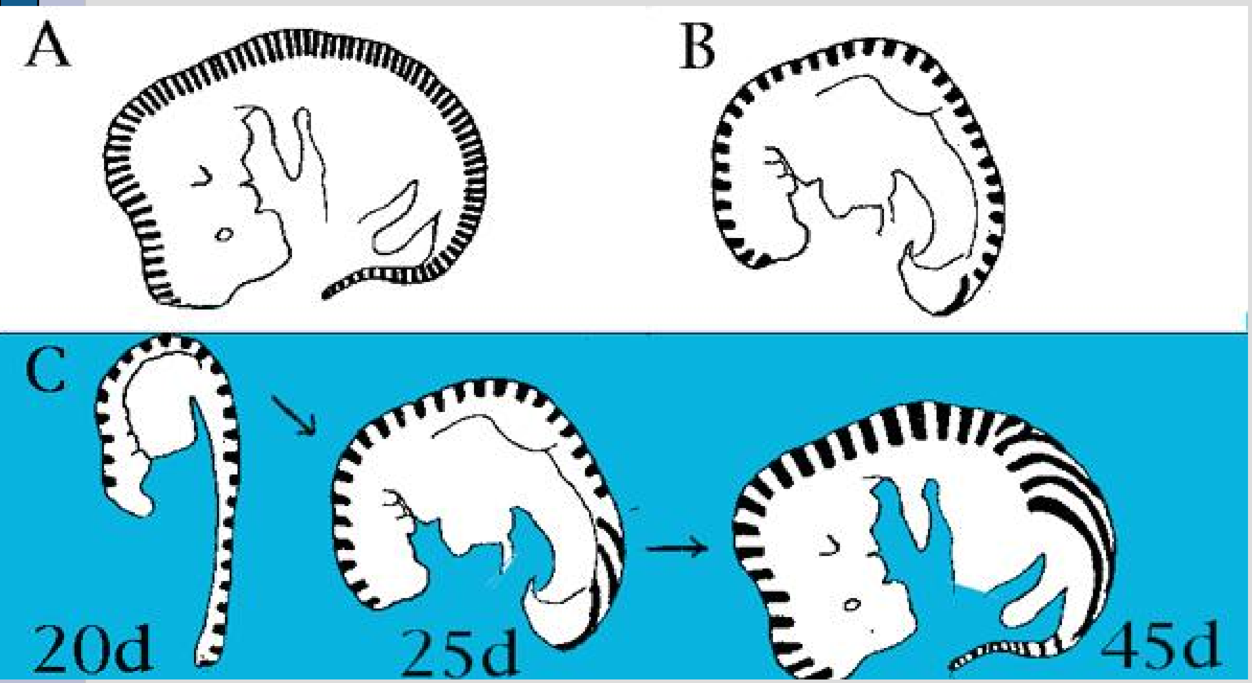
• negative feedback: it responds in opposite direction





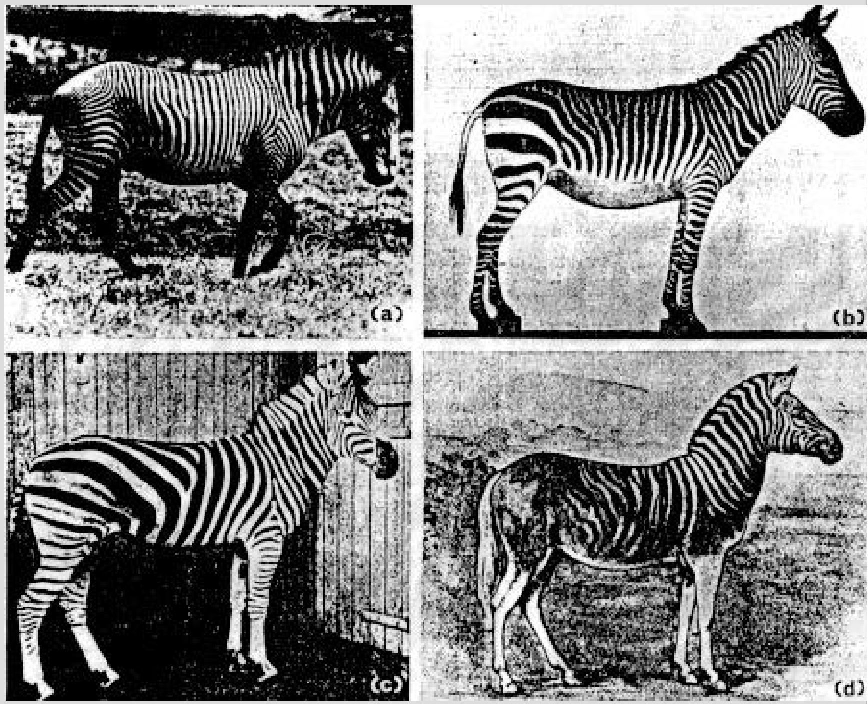
Zebras

Imperial Zebra: week 5 Mountain Zebra: week 4



Common Zebra: week 3

A B

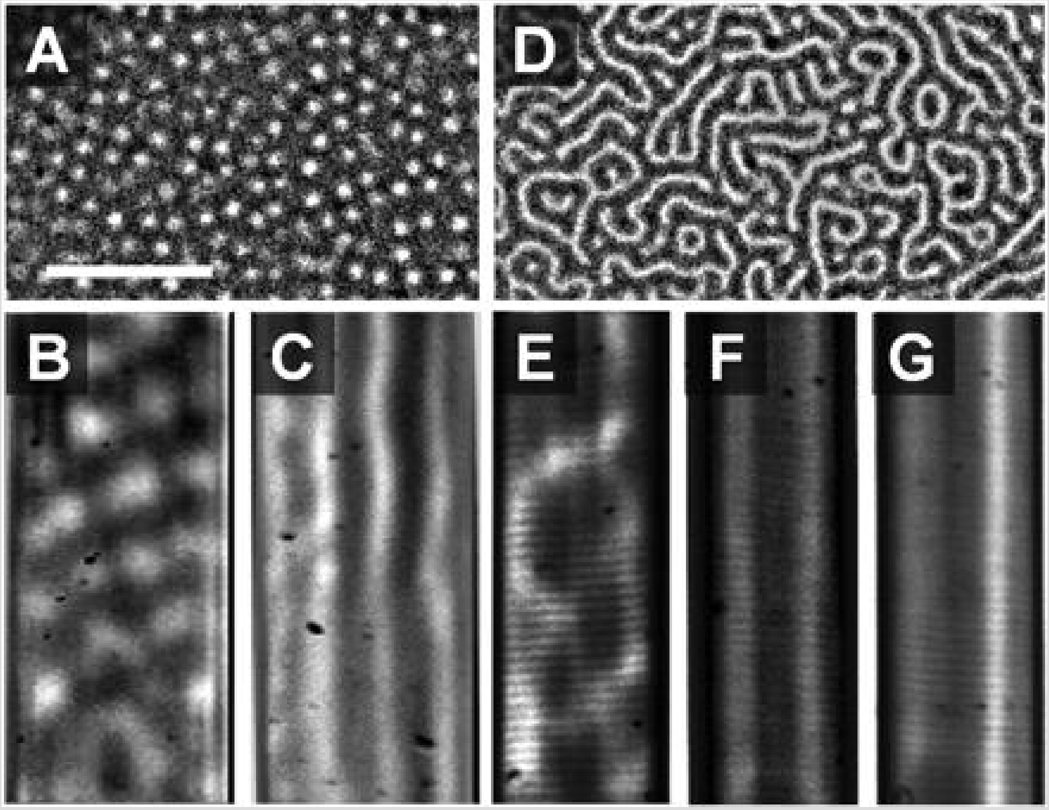


C D

(a) Imperial zebra (Equus grevyi)
(b) Mountain zebra (Equus zebra)
(c) Common zebra (Equus burchelli)
(d) Quagga (Equus quagga).

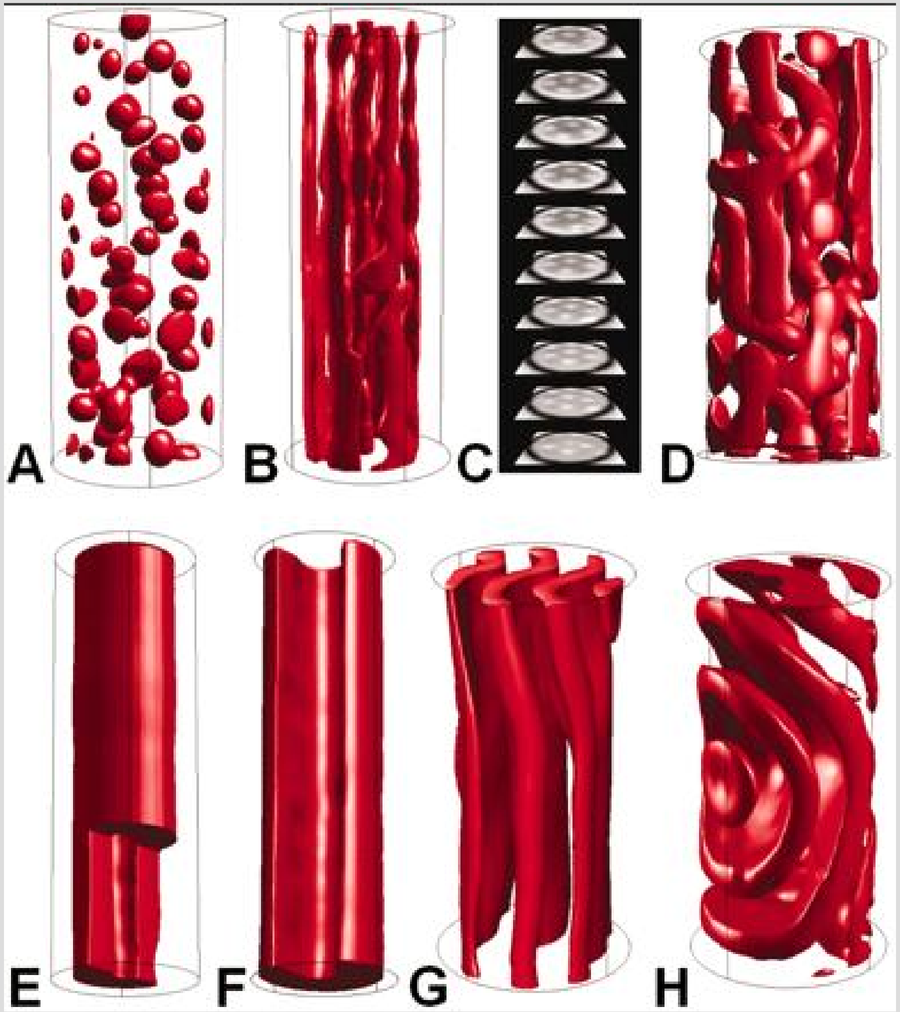
Bard 1977

Stop press: Turing patterns in 3 dimensions



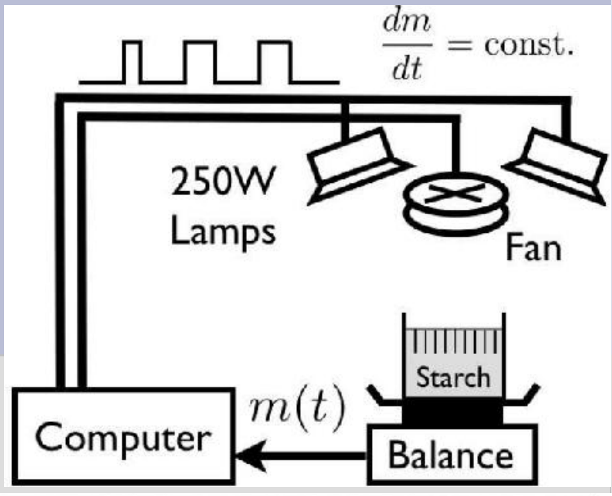
Snapshots of 2-d and 3-d patterns

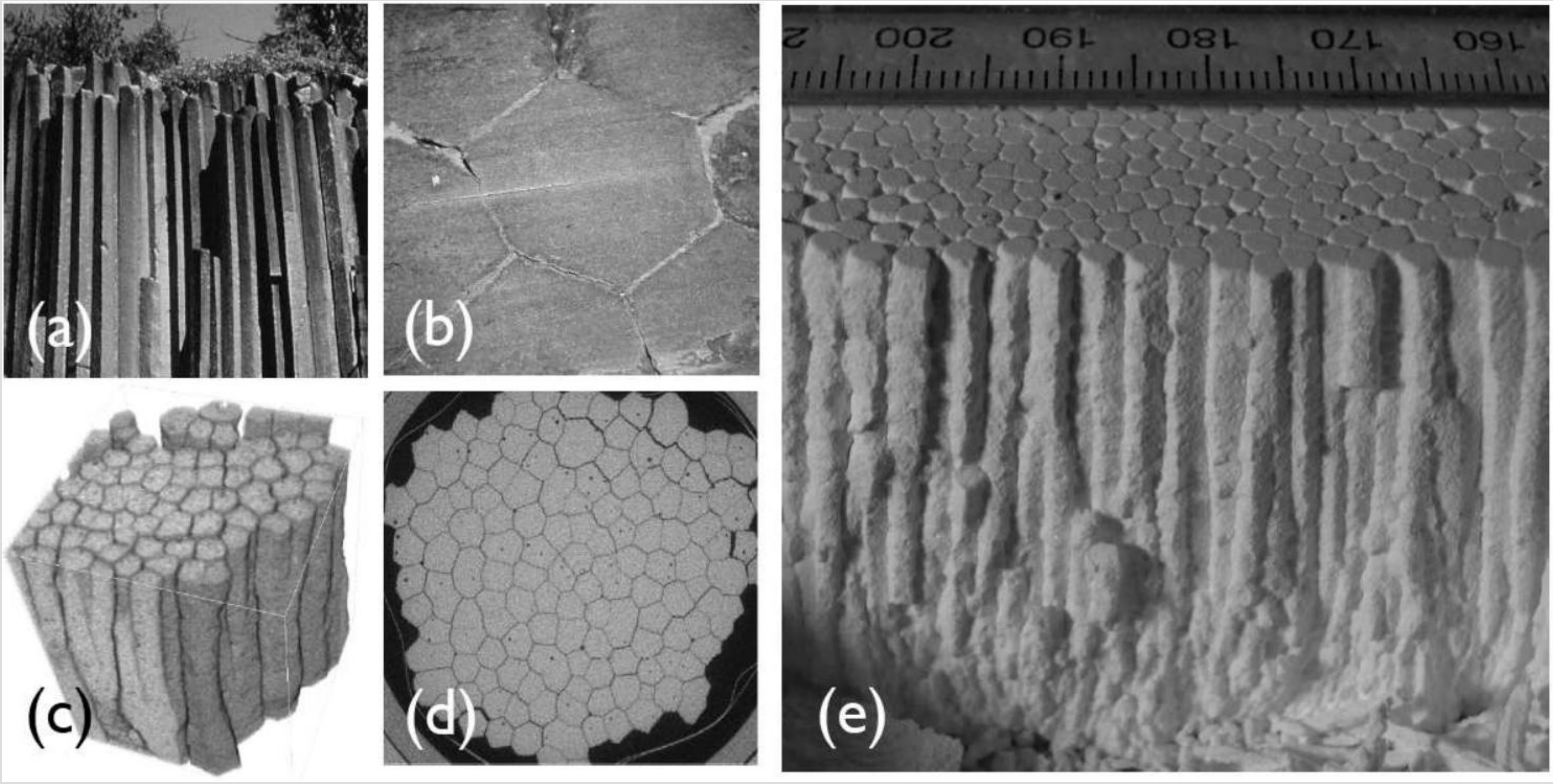
Bánsági, T., et al. (2011): Tomography of Reaction-Diffusion microemulsions reveals Three-Dimensional turing patterns. Science 331 (6022), 1309-1312.



Tomographically reconstructed concentration fields

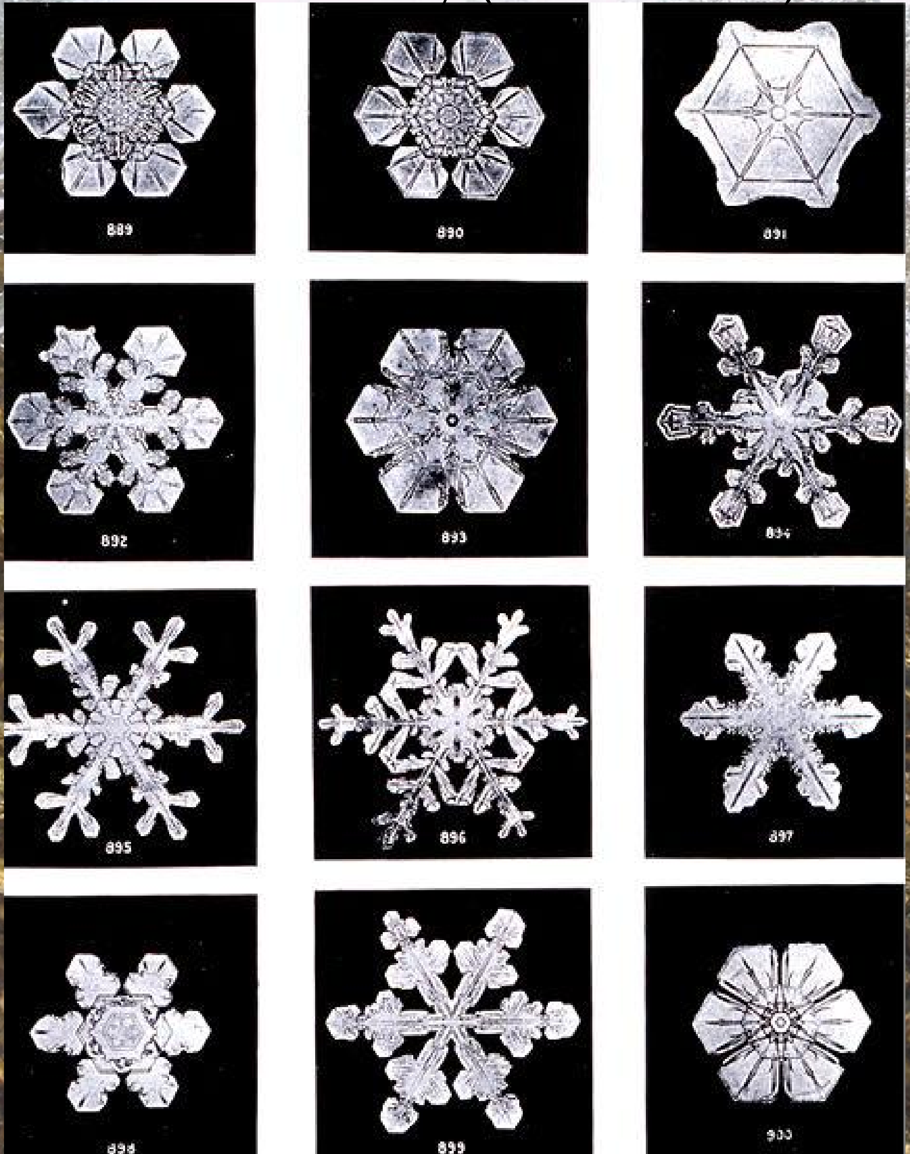
An experiment with cornstarch



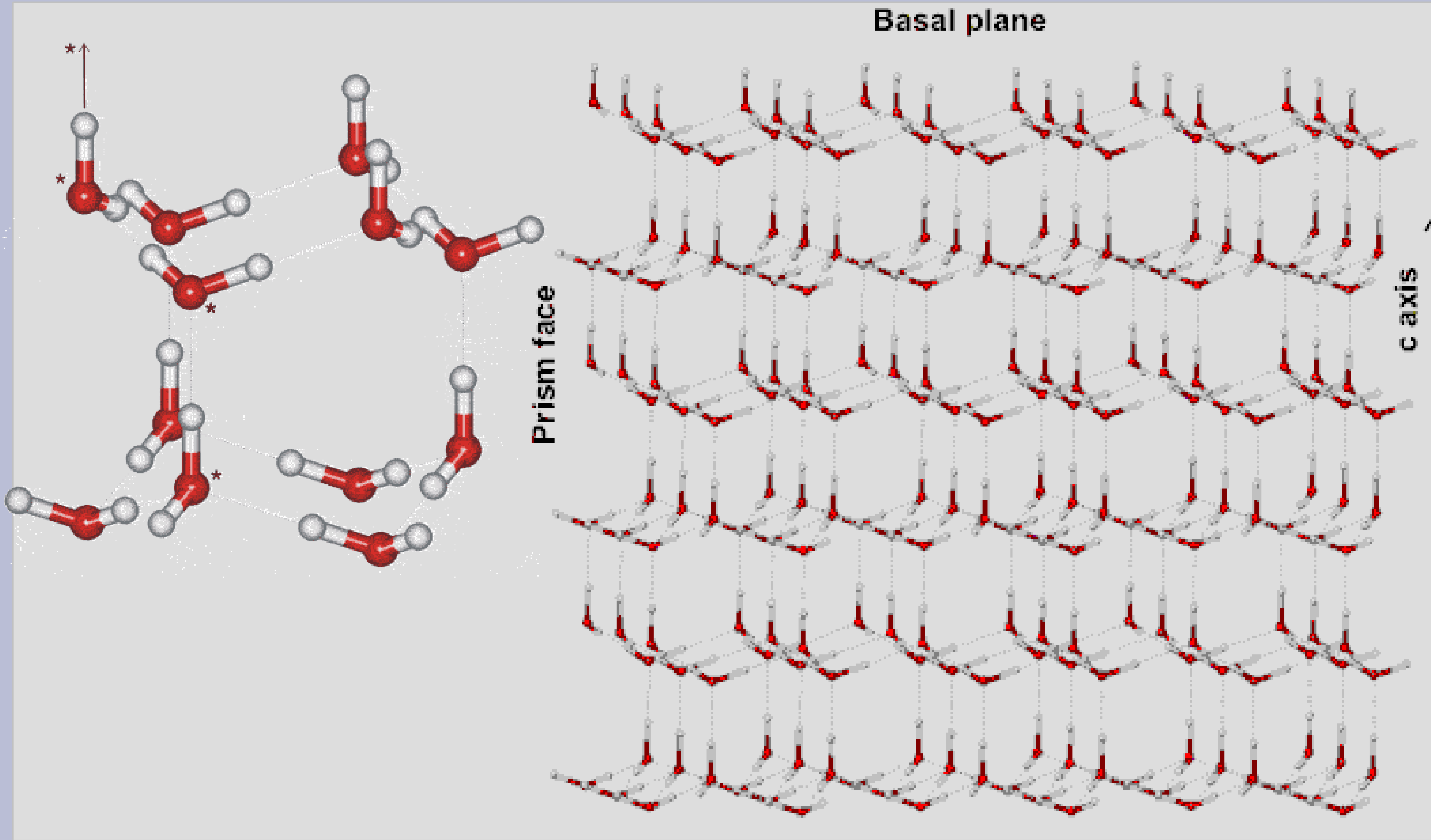


Snowflakes

Wilson Bentley (1865-1931)



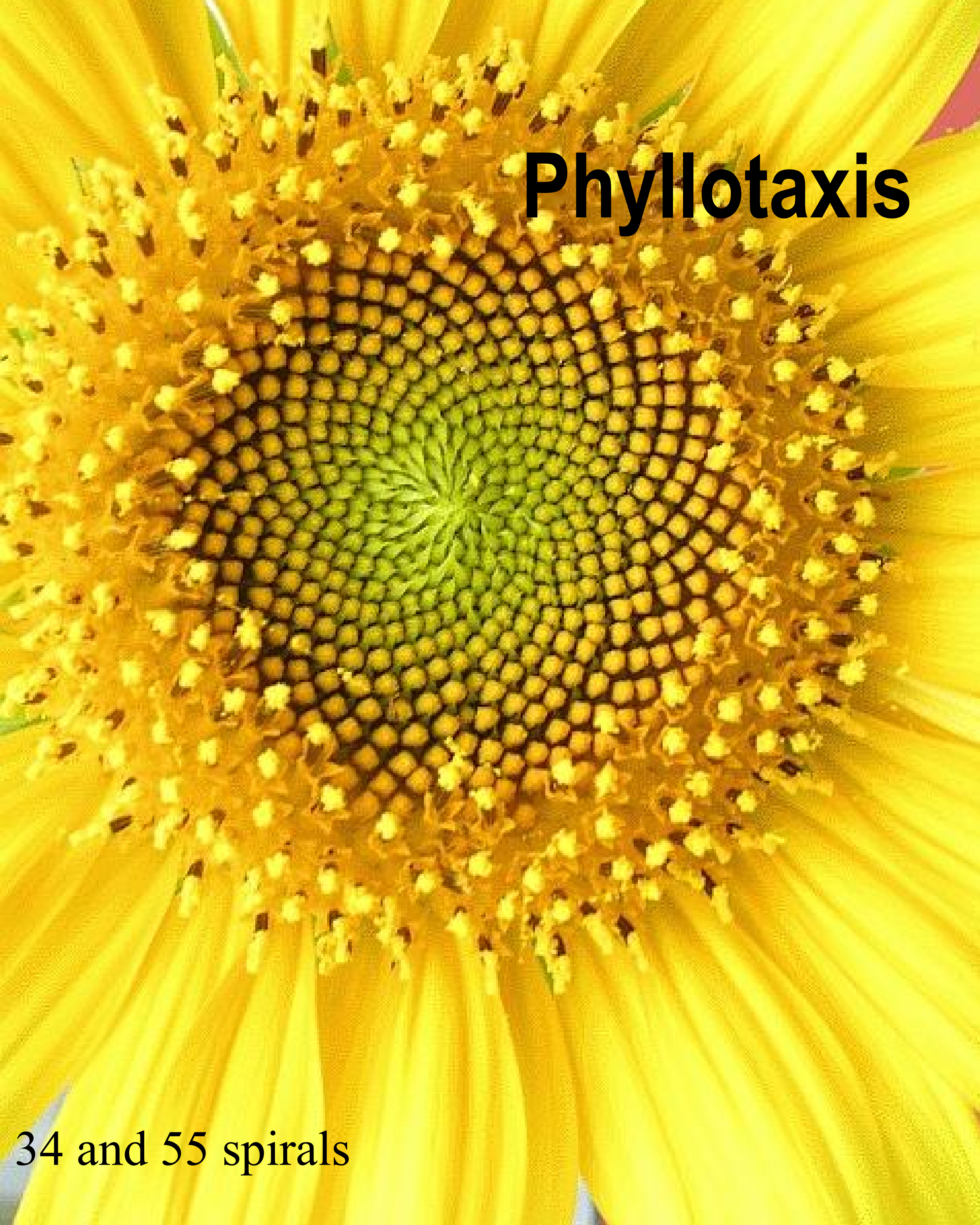
Water Ice (hexagonal Ice I_h)



Other fingering mechanisms

- Basic idea: Growth at the tip is easier than at the base (positive feedback!)
- A lot of different examples and names:
  - Diffusion limited aggregation (DLA): growth of dendrites in a solution (e.g. snowflakes)
  - Viscuous fingering; flow in porous medium (Saffman-Taylor instability)
  - Fingering in solidification (Mullins-Sekerka instability)

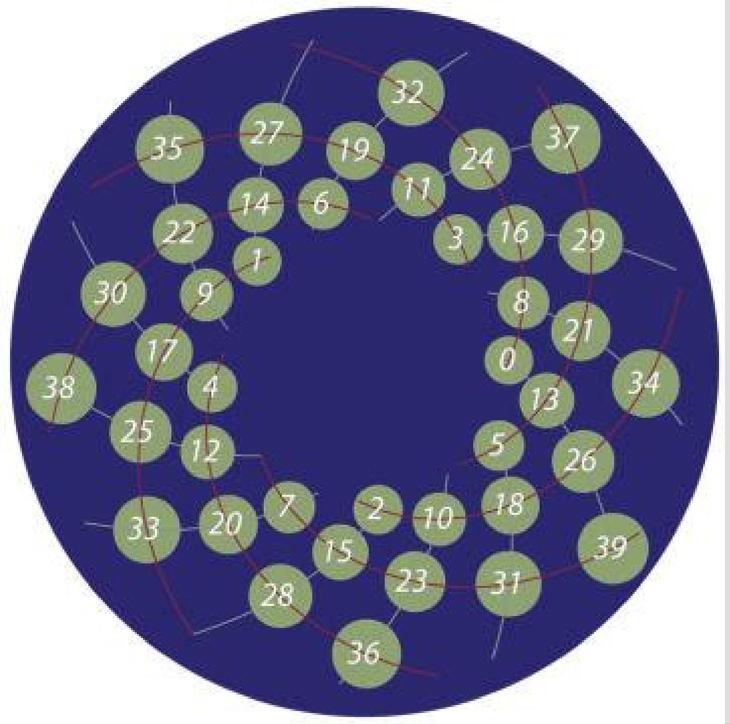
Phyllotaxis

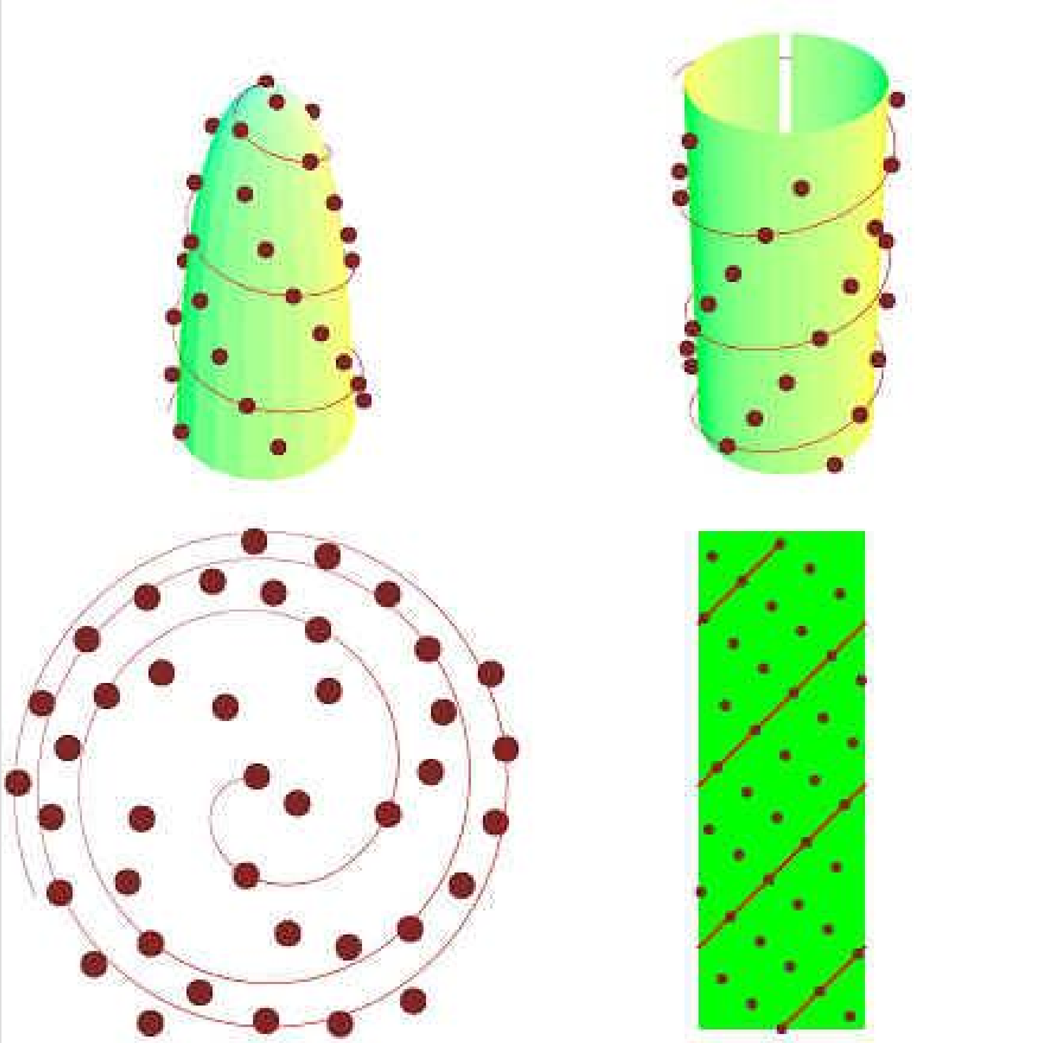


34 and 55 spirals

Two spirals...

- paristiche: spirals made up of next neighbours
- **generative spiral**





The most irrational number

There is a „most irrational“ number, and it turns out (surprise, surprise) the golden number.

...

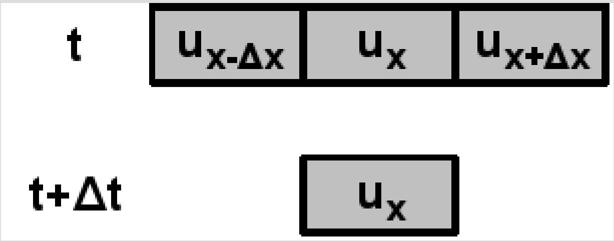
It is the most badly „approximable-by-rational“ number there is!

Its „badness“ is exceeded only by the awkwardness of the preceding sentence.

(Adam, Mathematics in Nature, p220)

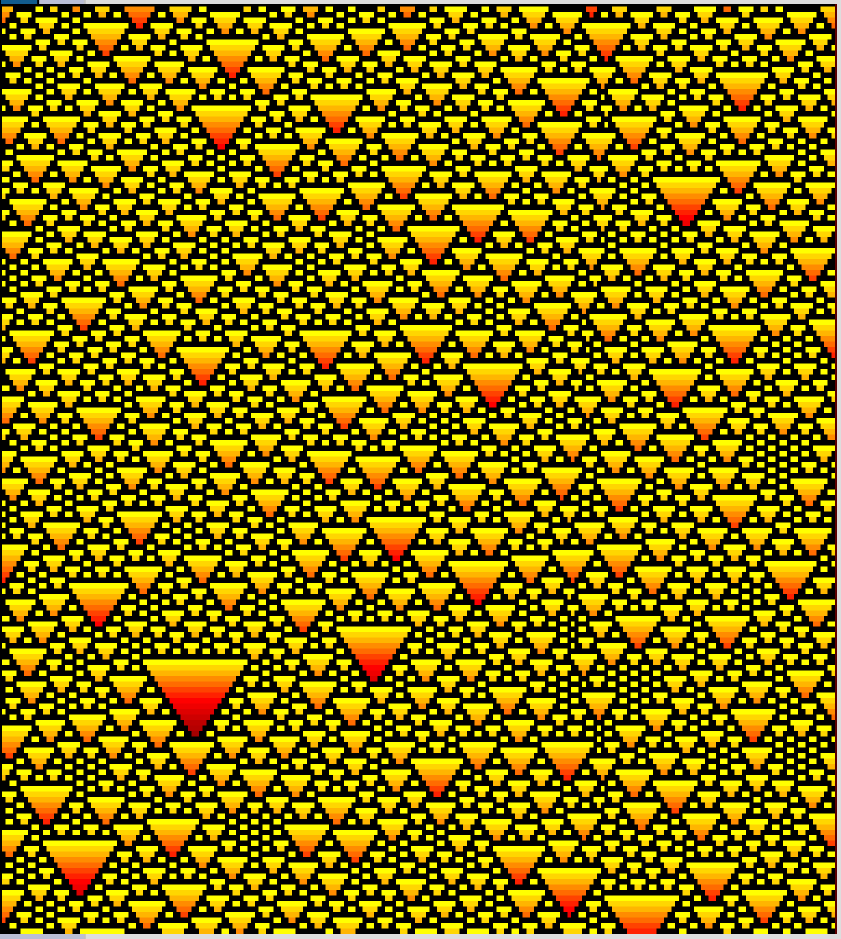
(1-dimensional) Cellular automata

- Computer model
- The model is "discrete":
  - Array (line) of cells
  - Each cell can have a finite number of different states ("alive" or "dead")
- State of a cell changes each timestep depending on the state of its neighbours



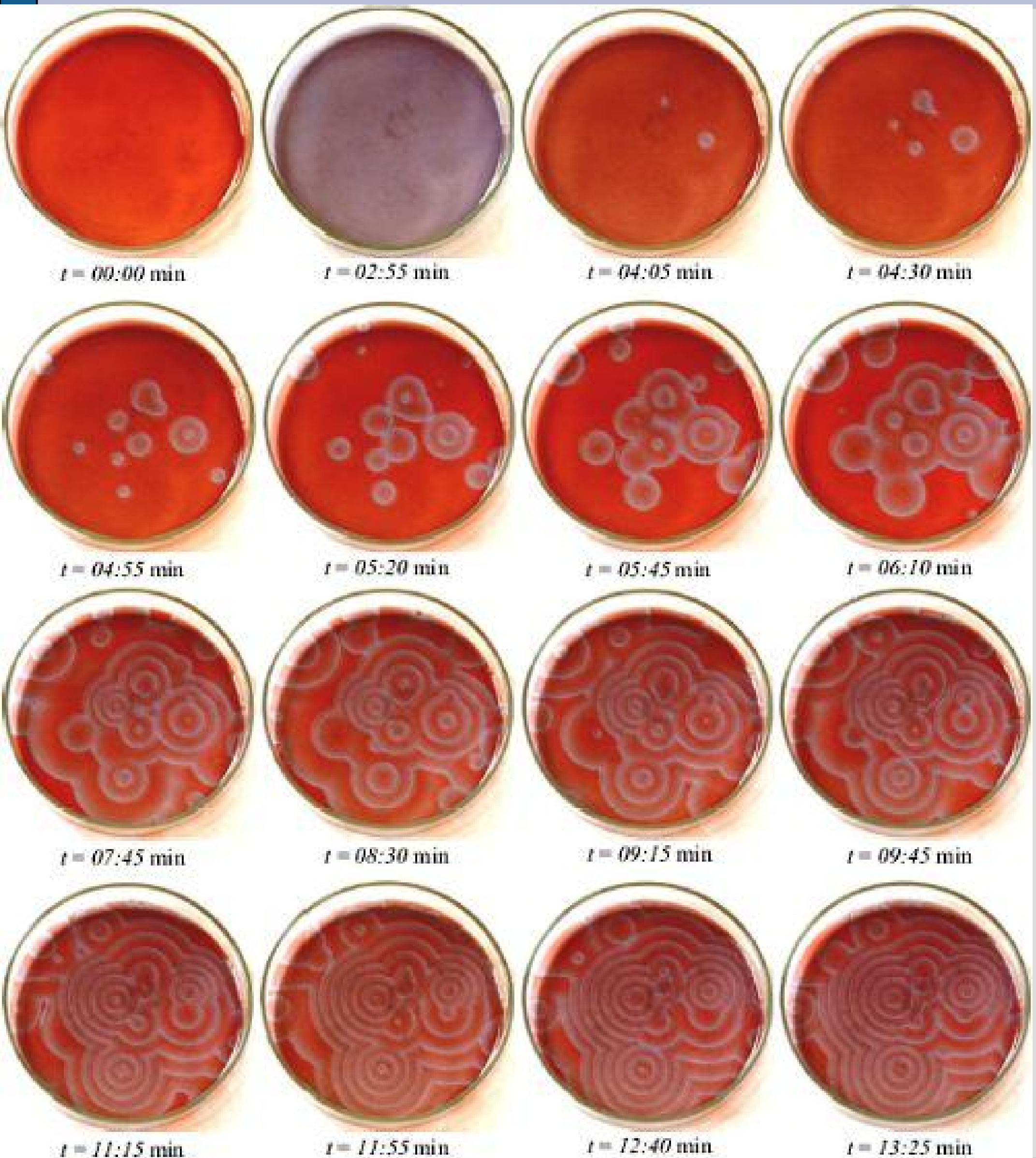
Some more 1-d automata

- “Porridge” (R1,C0,M1,S0,S3,B0,B2)

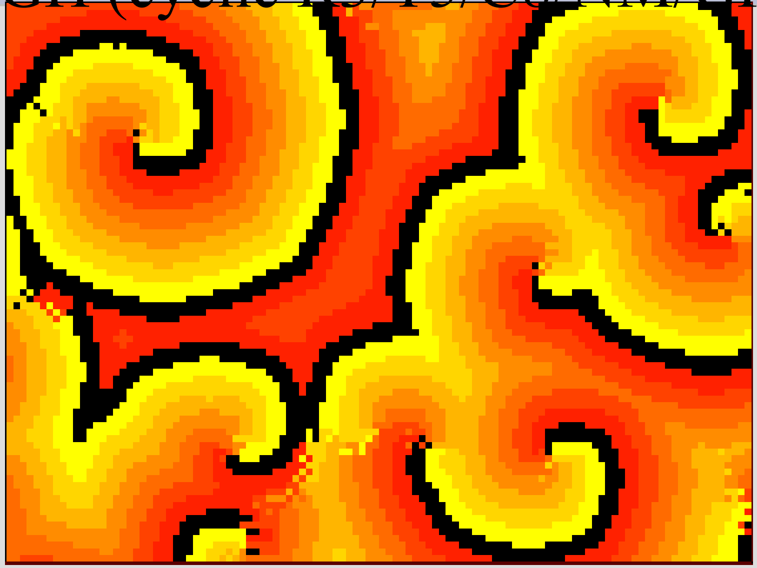


Conus textile

The Belousov-Zhabotinski reaction



GH (cyclic R3/T5/C8/NM/GH

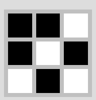


BelZhab (Gen 23/23/8)

Conway's “Game of Life”

- Still:
  - “Blocks”

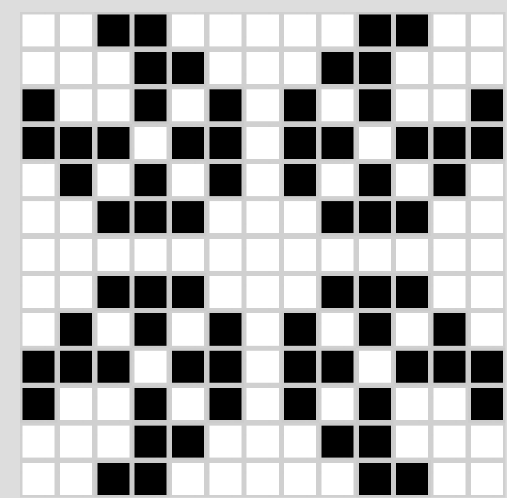
  - “boats”



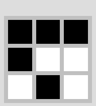
- Stationary 2-phase oscillators:
  - “blinkers”

  - “toads”

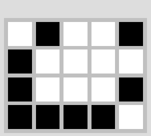
- 3-phase oscillator:
  - “pulsar”



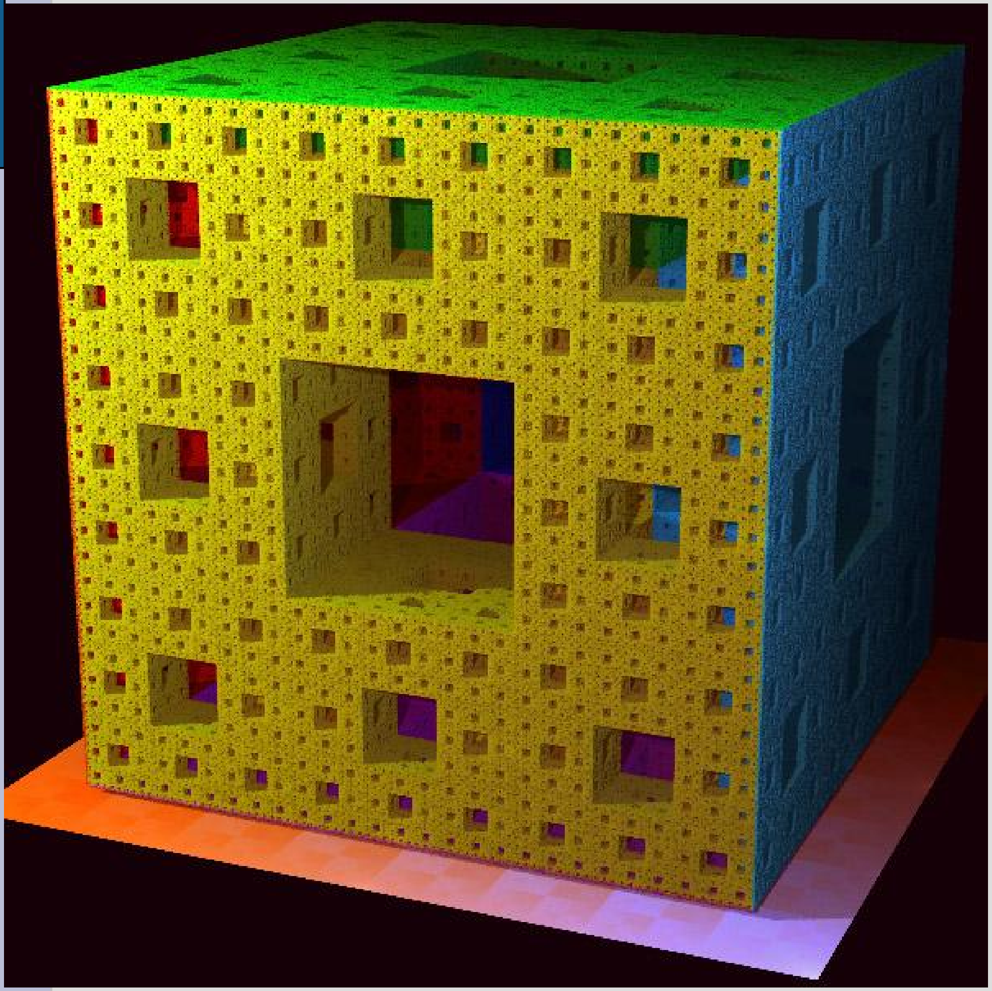
- Moving:
  - “gliders”

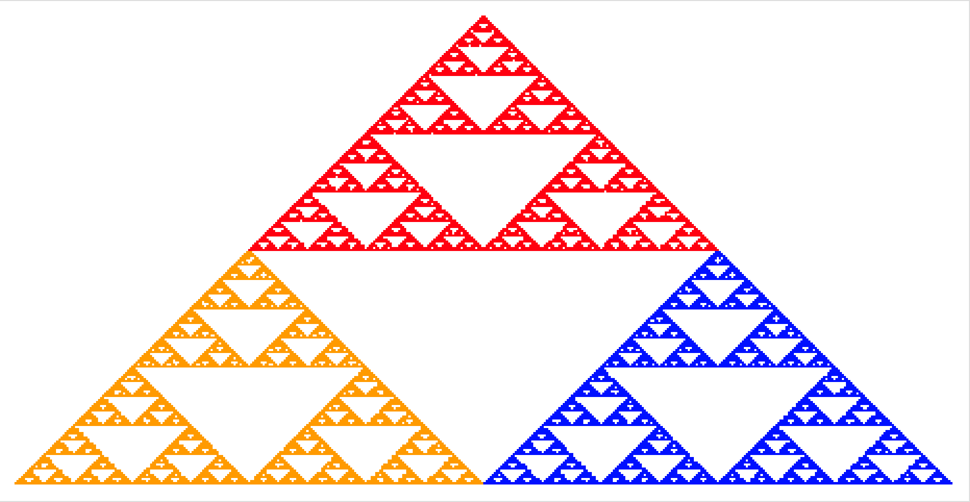


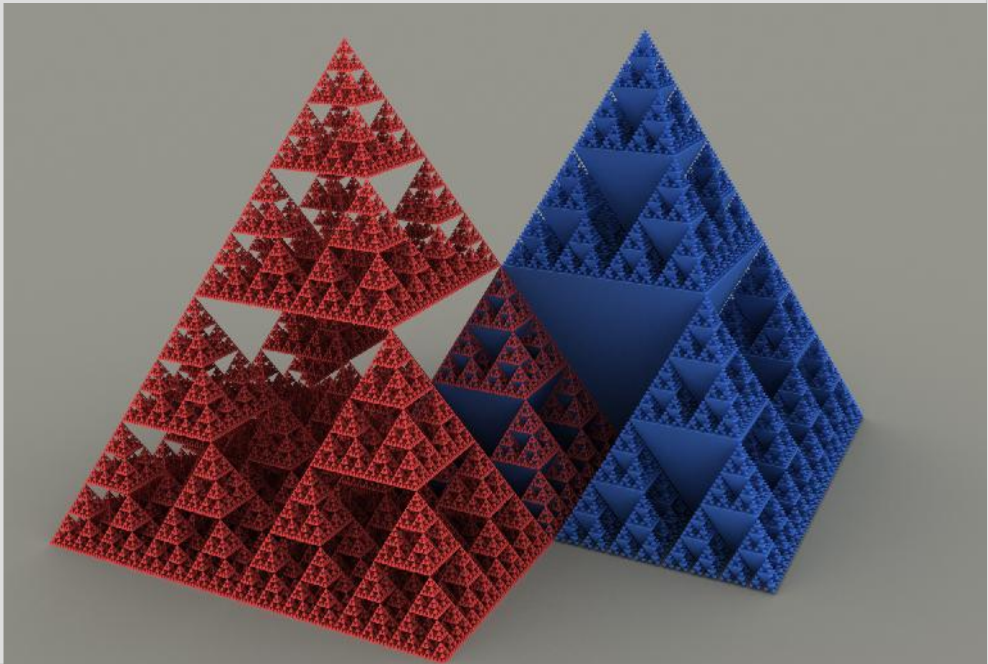
  - “lightweight spaceship”



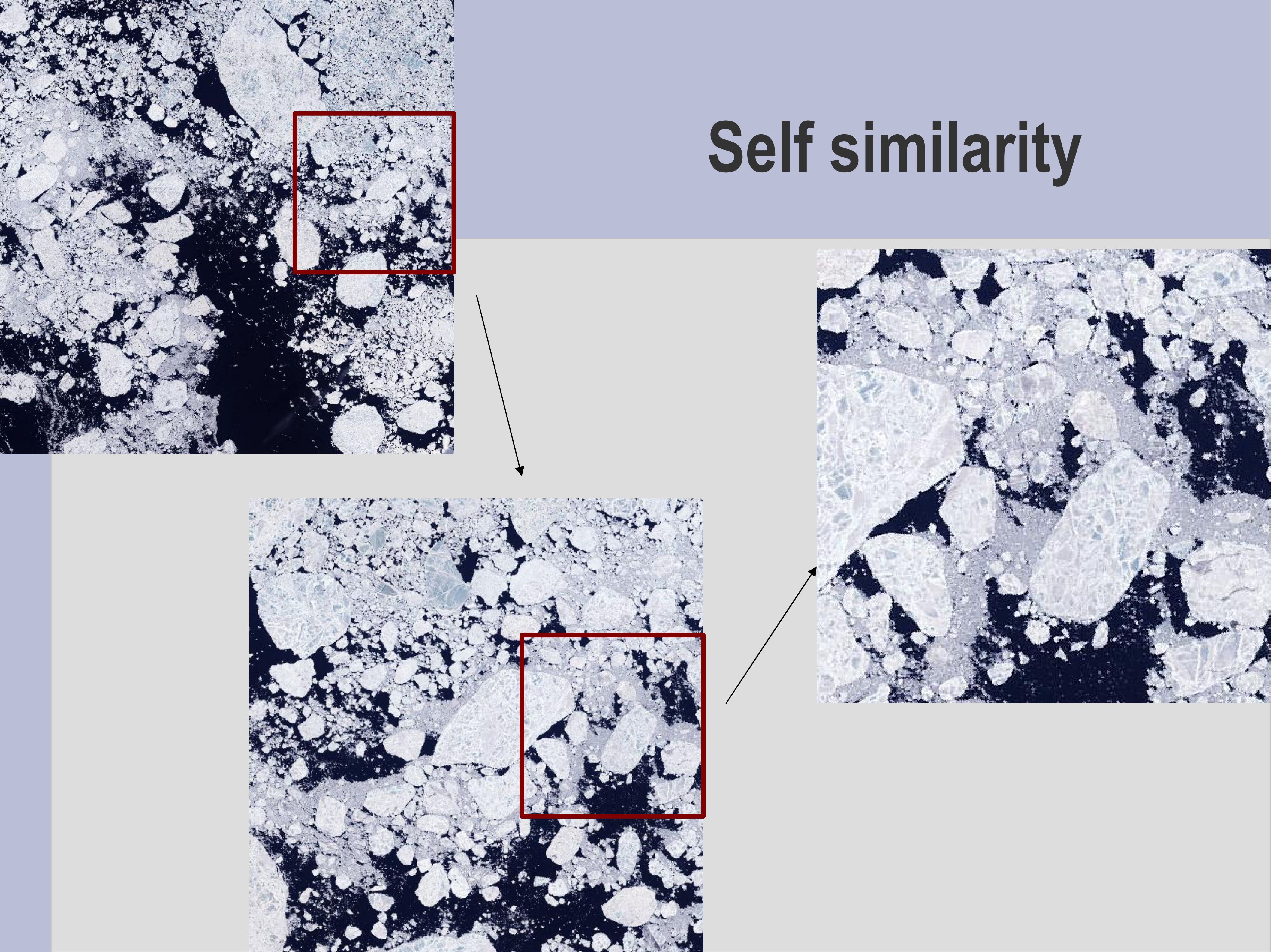
Menger sponge and Sierpinski pyramid





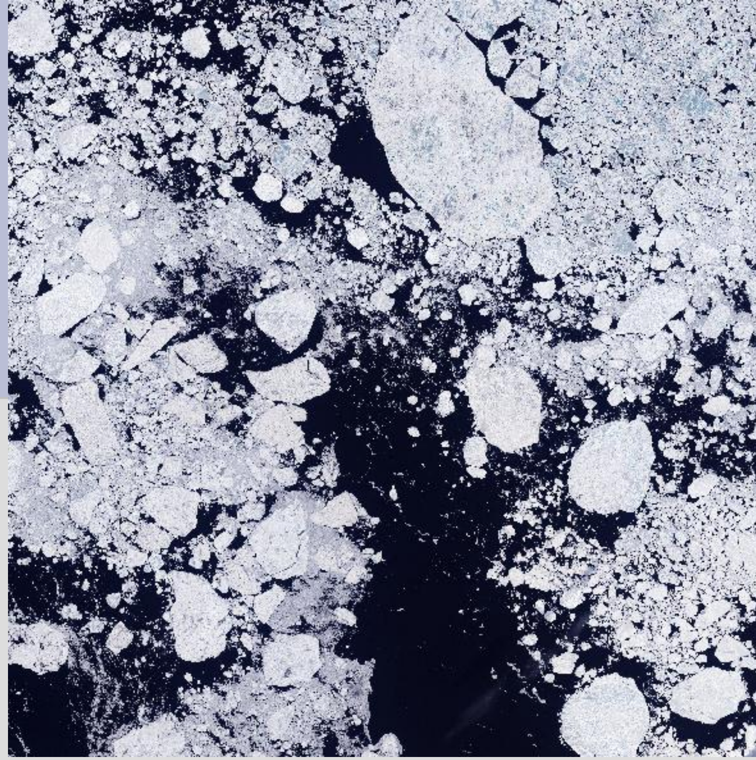


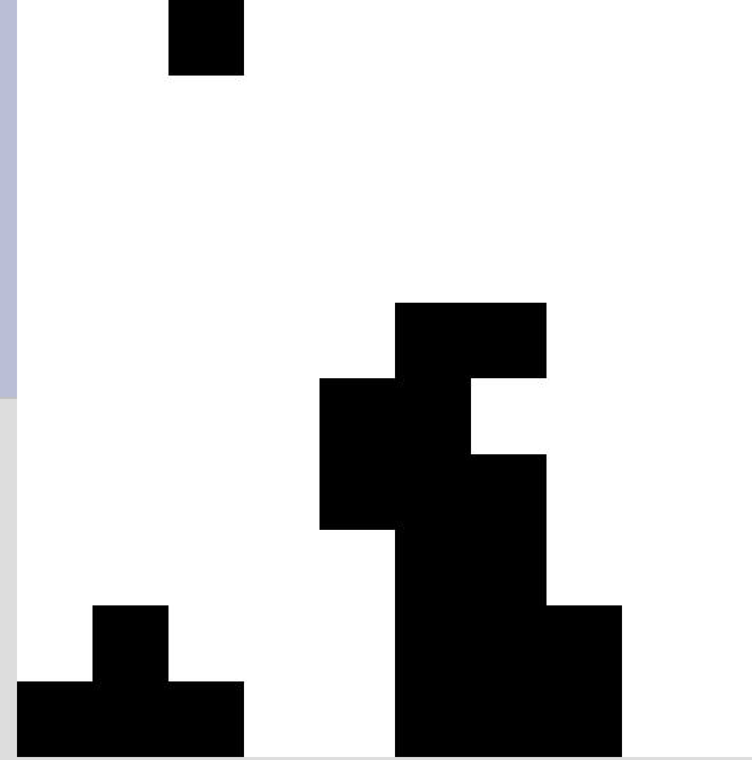
Self similarity



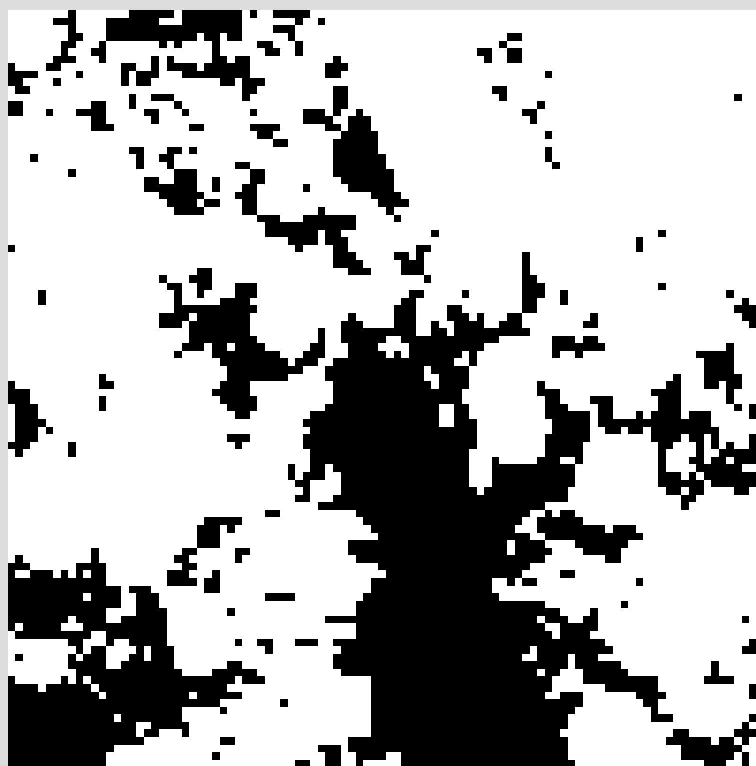
Full image:
6,975,486
/ 10,614,564

= **65.7%** ice





80 ice pixels out of 100 = **80%** ice cover



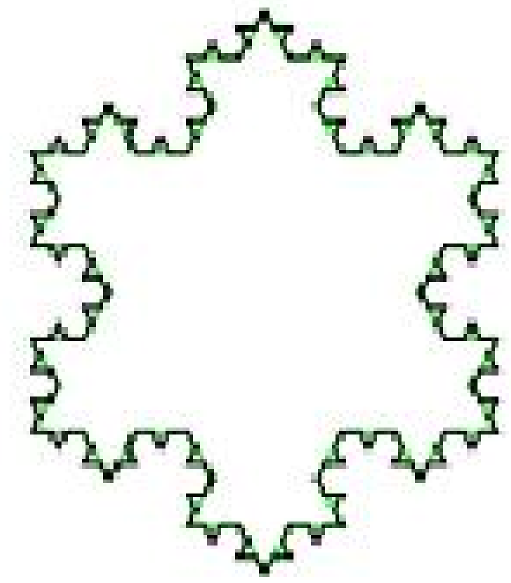
Dimension (using the
10x10 and 100x100 images):

a = 10
N = 7010/80 = 87.625

dim = log (N) / log (a) = 1.94

7010 ice pixels / 10000 in total
= **70.1%** ice cover

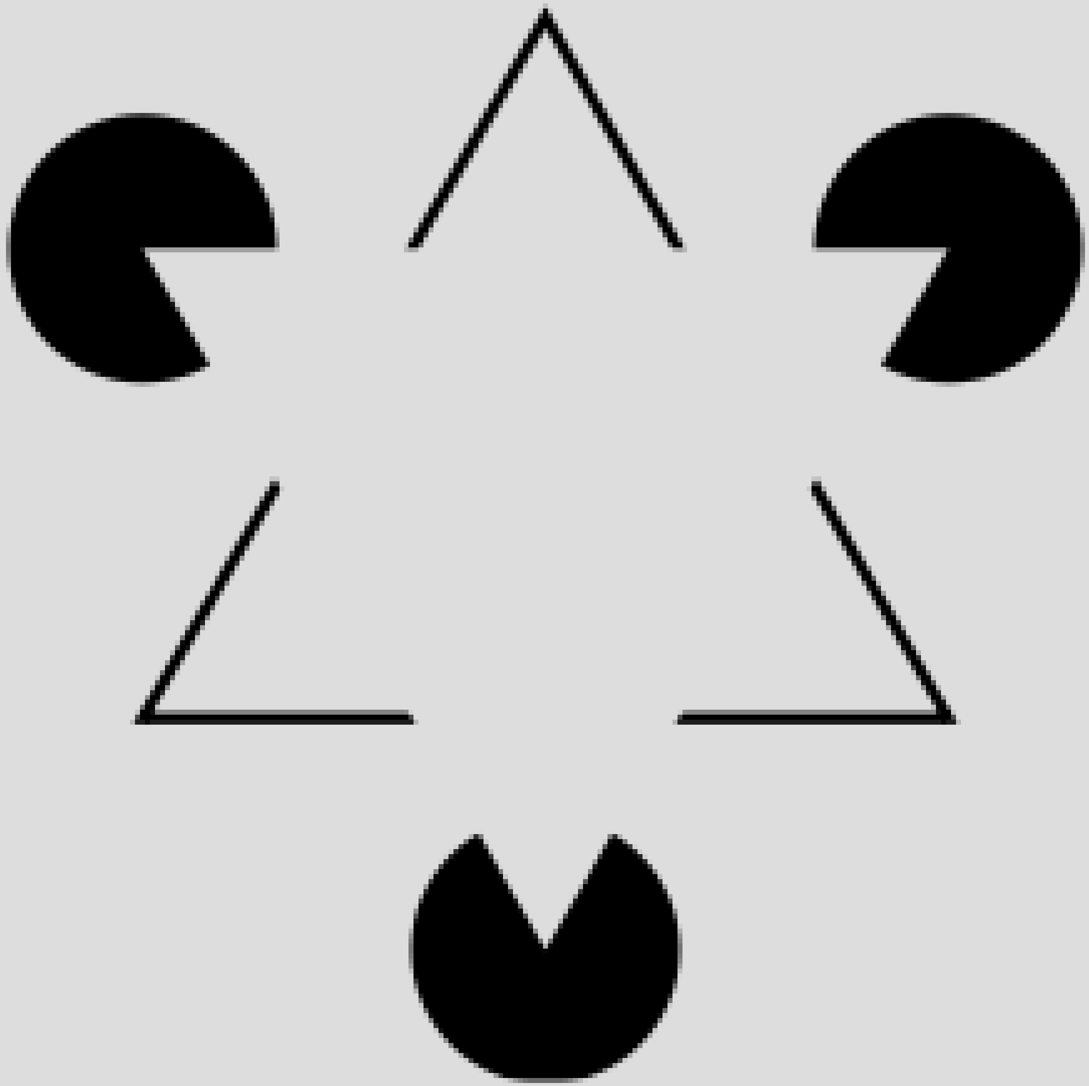
Frequent features of fractals



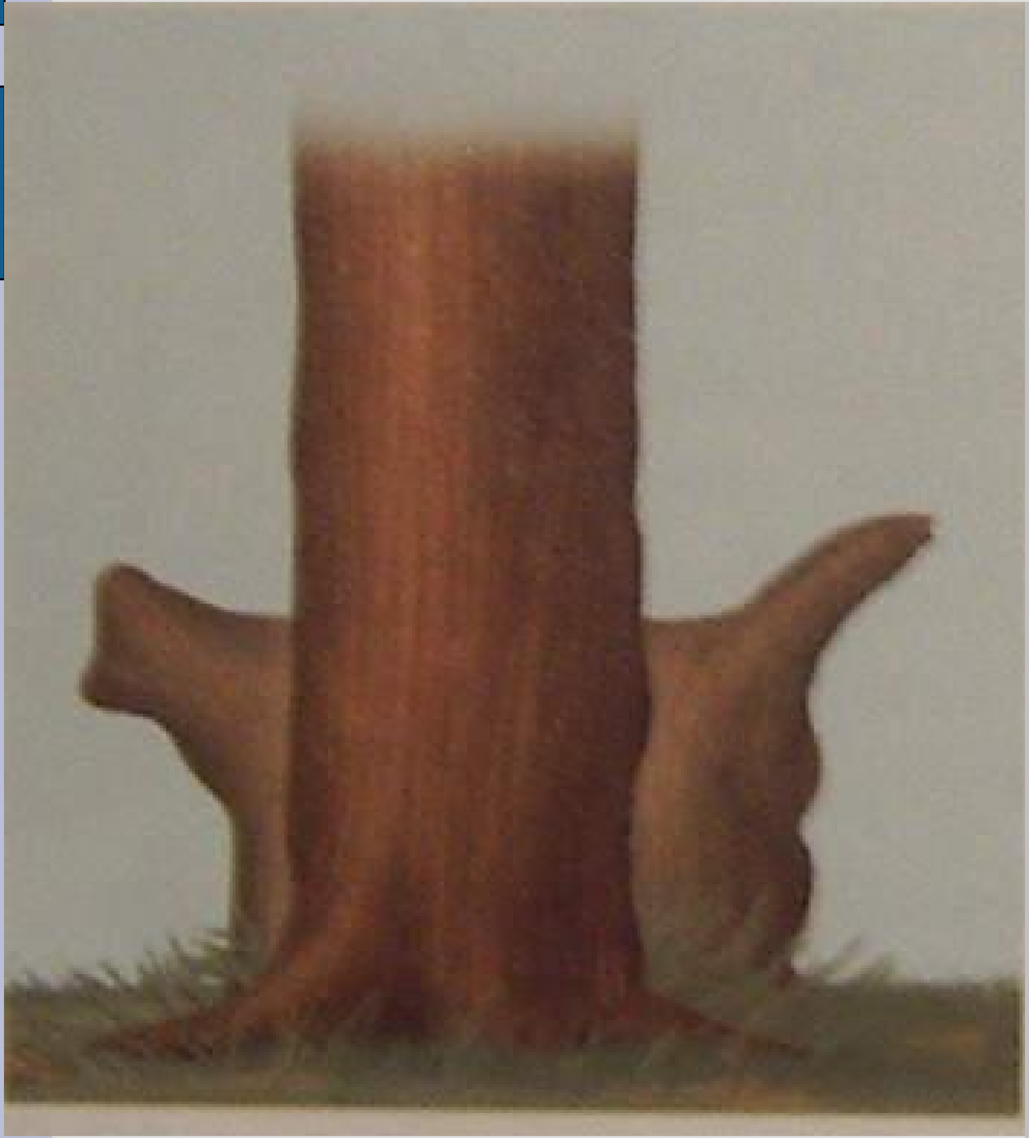
- F is **self-similar** (at least approximately or stochastically).
- F has a **fine-structure**: it contains detail at arbitrary small scales.
- F has a simple definition.
- F is obtained through a **recursive** procedure.
- The geometry of F is not easily described in classical terms.
- It is awkward to describe the local geometry of F.
- The size of F is not quantified by the usual measures of length (this leads to the **Hausdorff dimension**)

(after Falconer 1990)

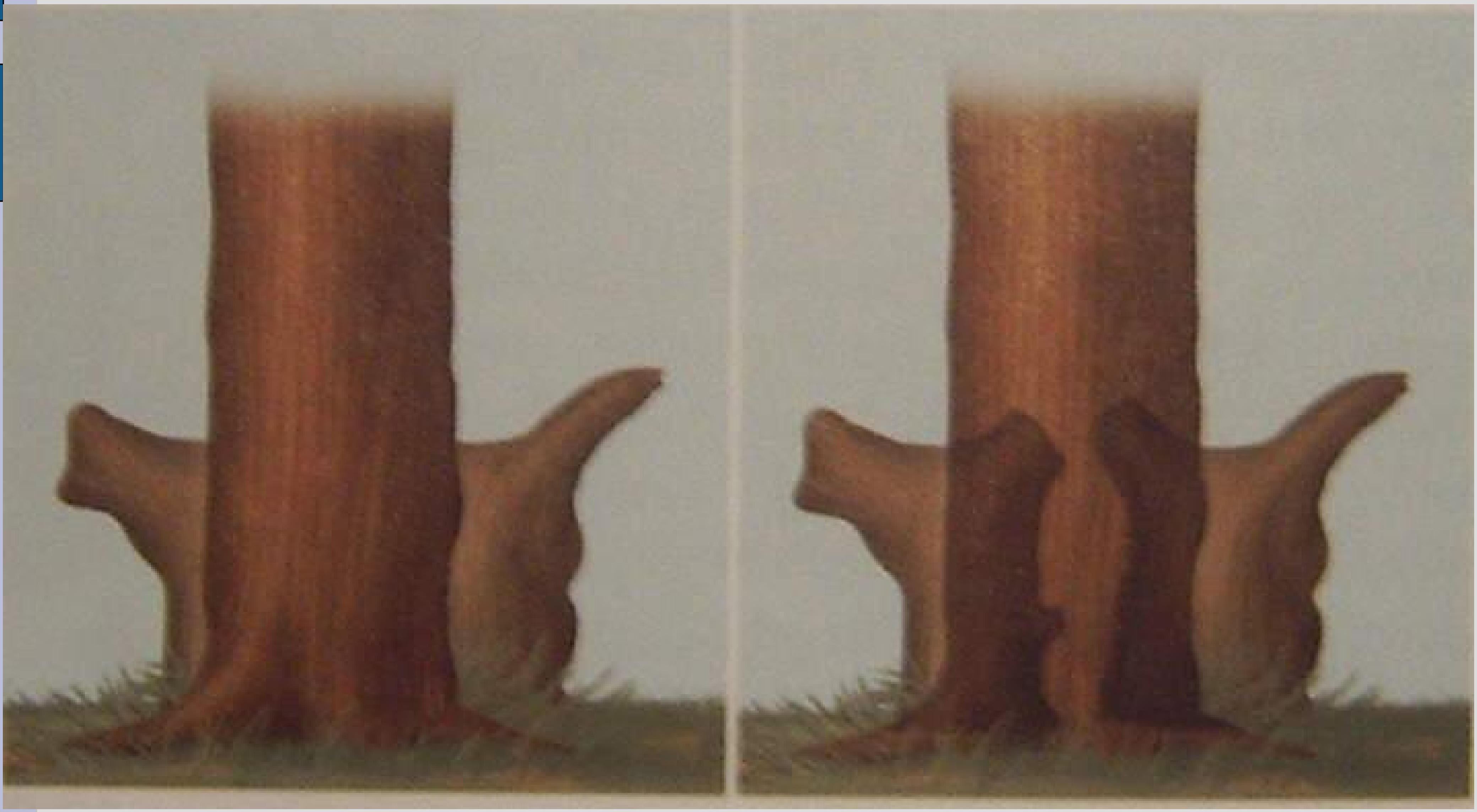
Perception: Kanizsa figures



Gestalt perception



Why?
Ambiguity in the real world



Different levels of description

- Mechanisms:
  BZ reaction, pheromones (social insects), genetics, ...
- Phenomenological description
  excitation wave, fractal, chaos, ...
- Modelling
  cellular automata, boids (swarming behaviour), differential equations, ...

=>

- Need many levels of description
- No simple „one size fits all“ theory