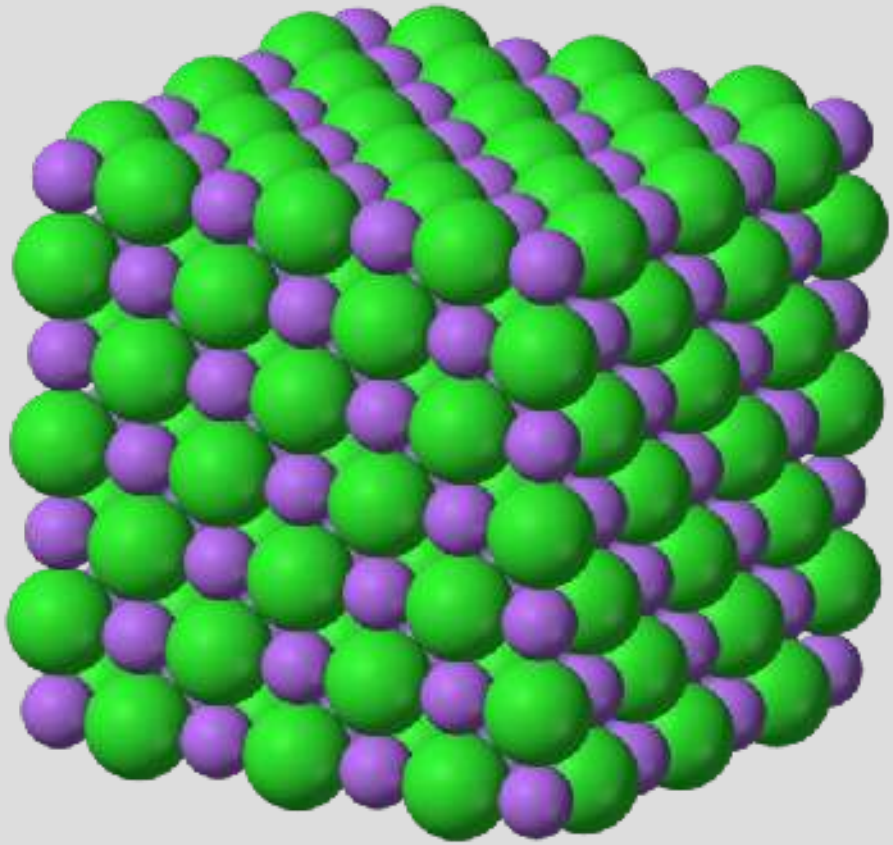
Patterns in Nature 6
Growth processes

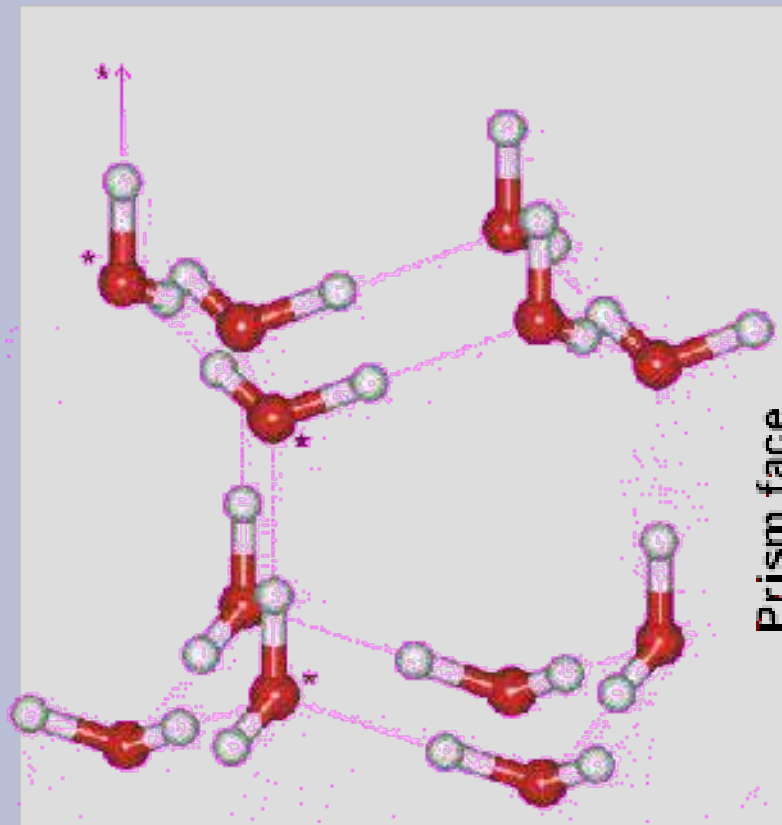
Stephan Matthiesen

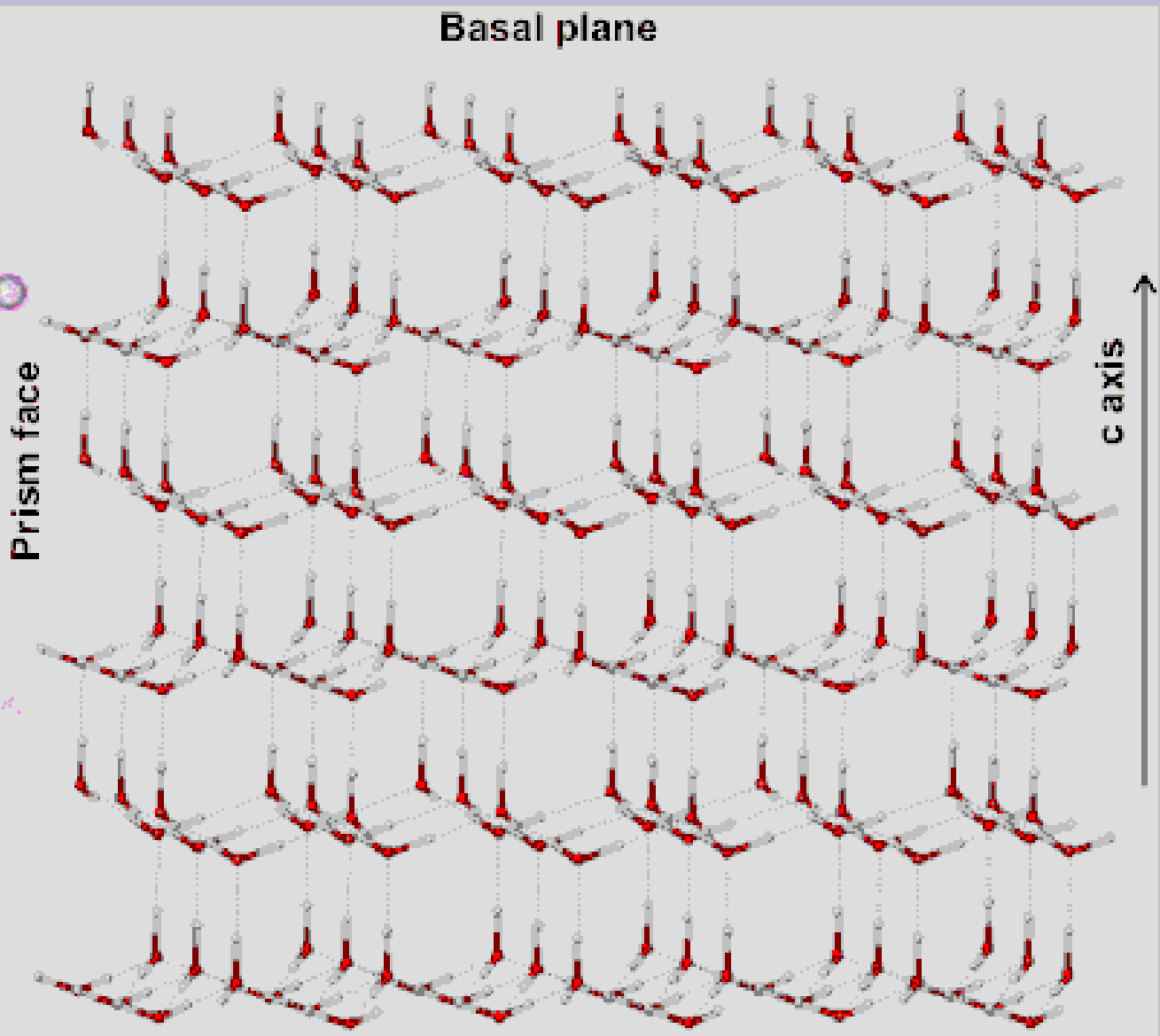
Crystals

Salt (Sodium Chloride)

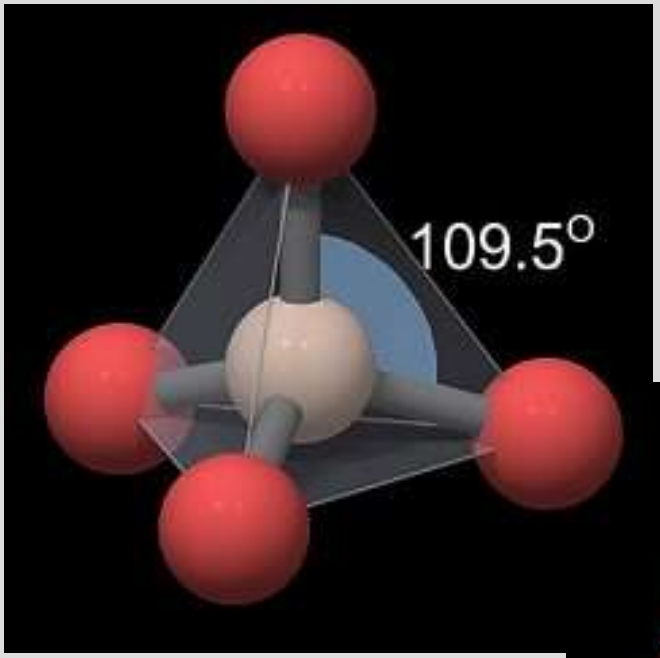


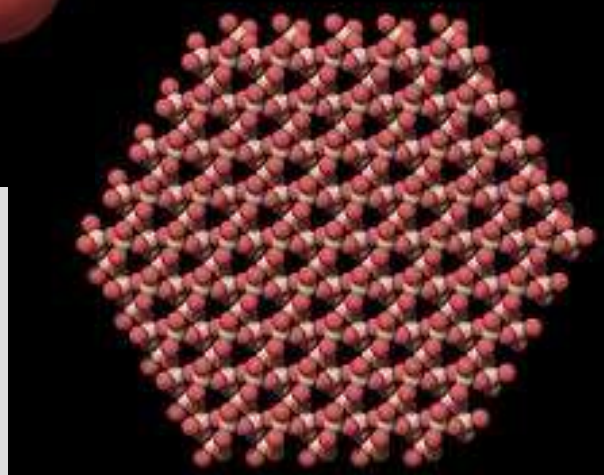
Water Ice (hexagonal Ice I_h)

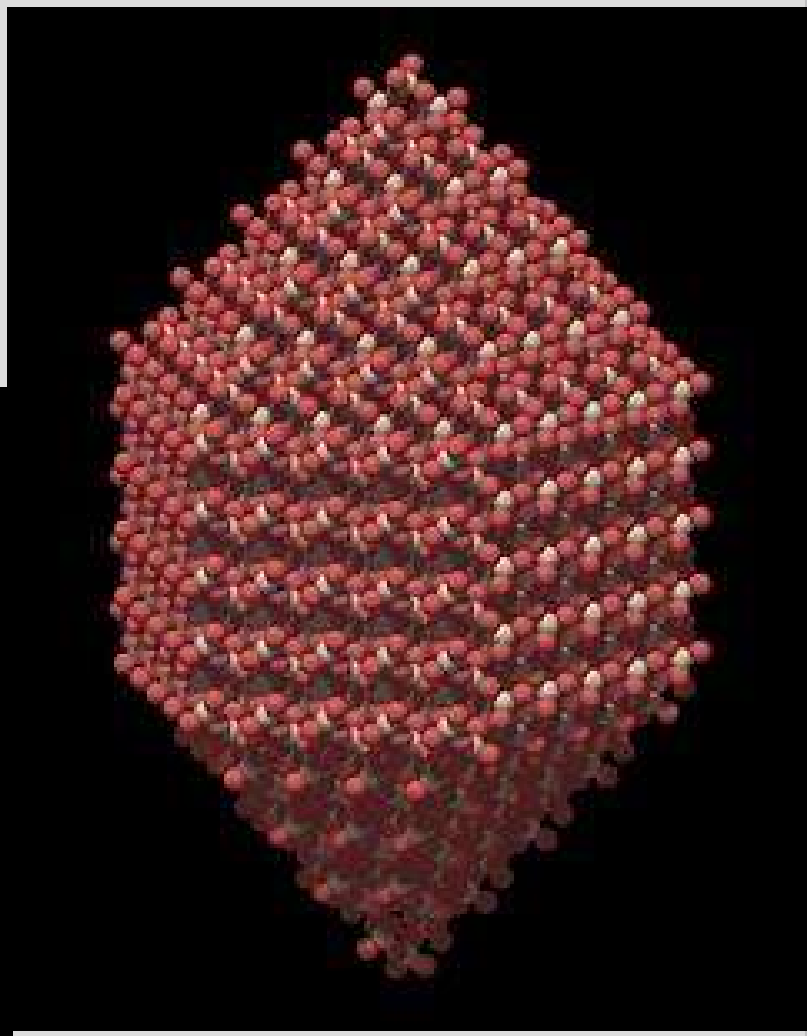




Quartz (SiO_2)



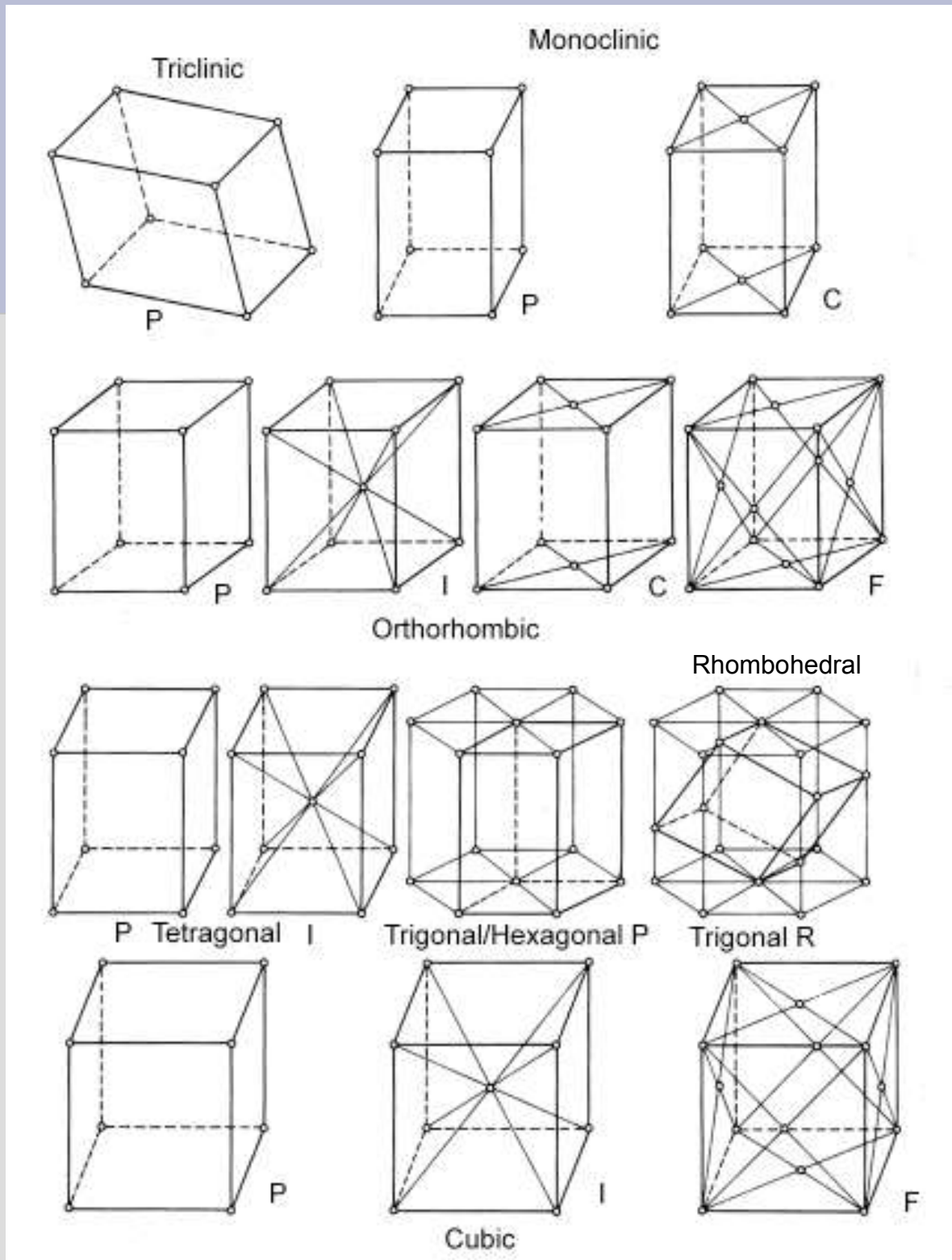


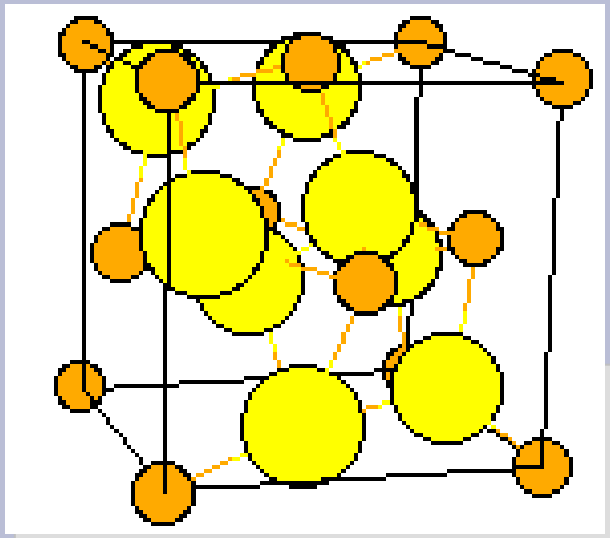


http://www.quartzpage.de/gen_struct.html

Crystal Systems

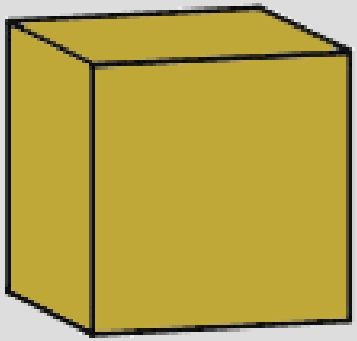
- 7 Crystal Systems
- 14 Bravais lattices
- 230 space groups

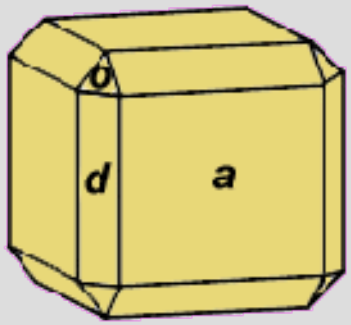


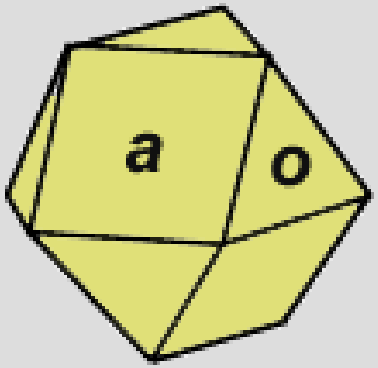


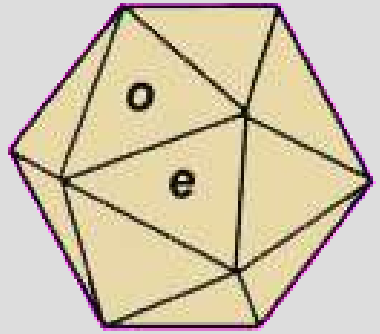
Crystal Forms of Pyrite (cubic)

Pyrite = FeS_2 = Fe^{2+} + S_2^{2-}



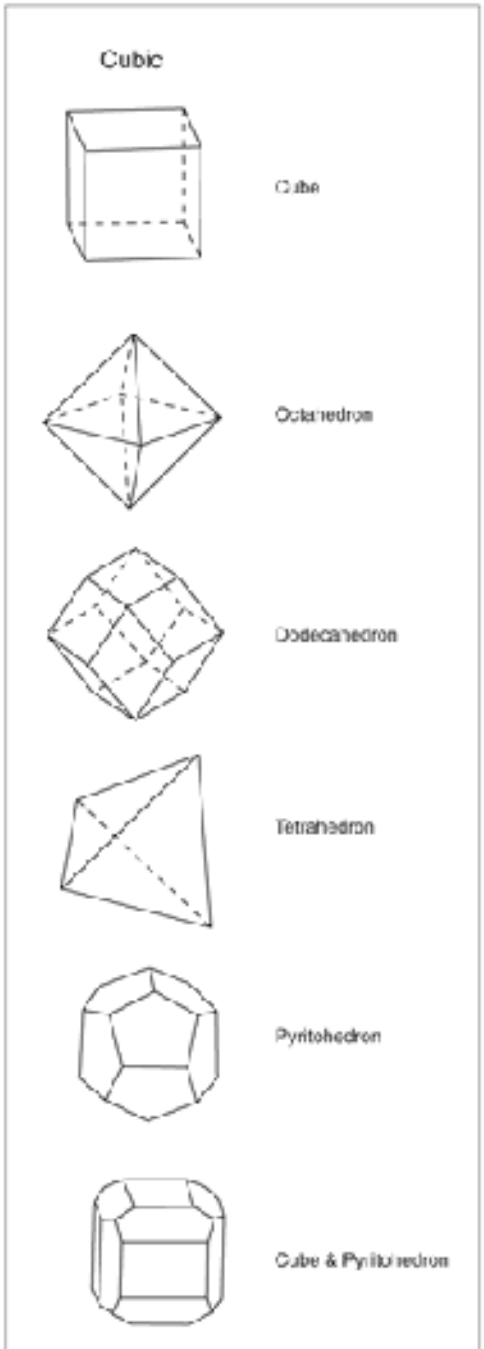


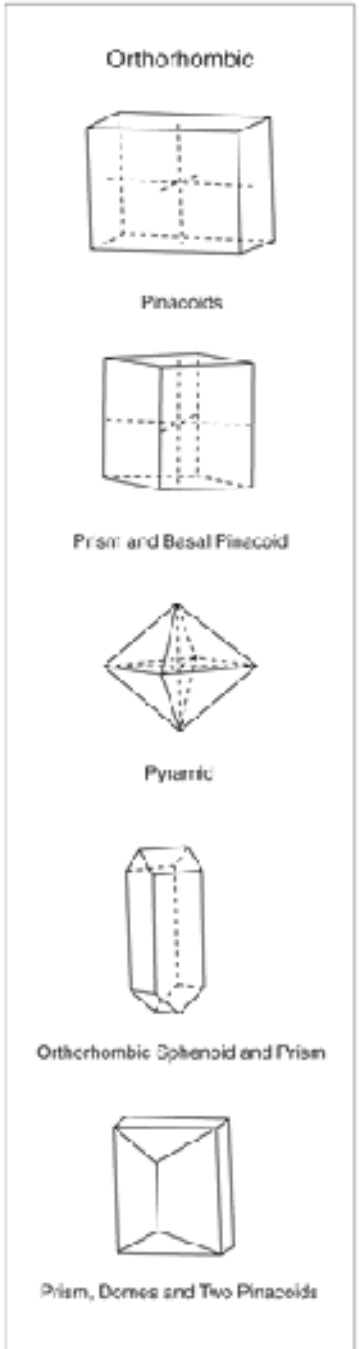


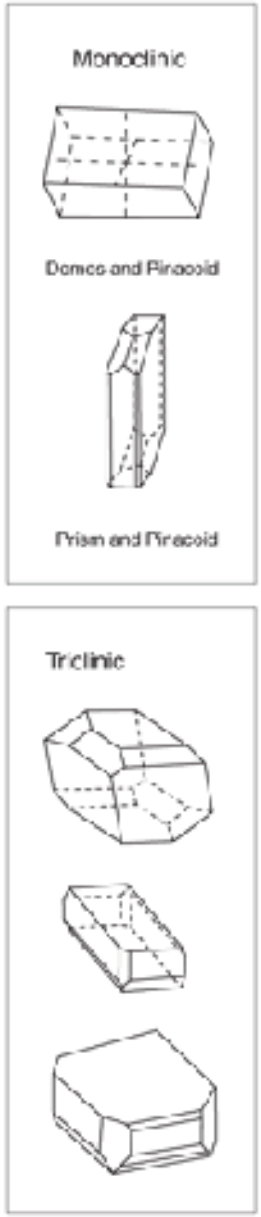


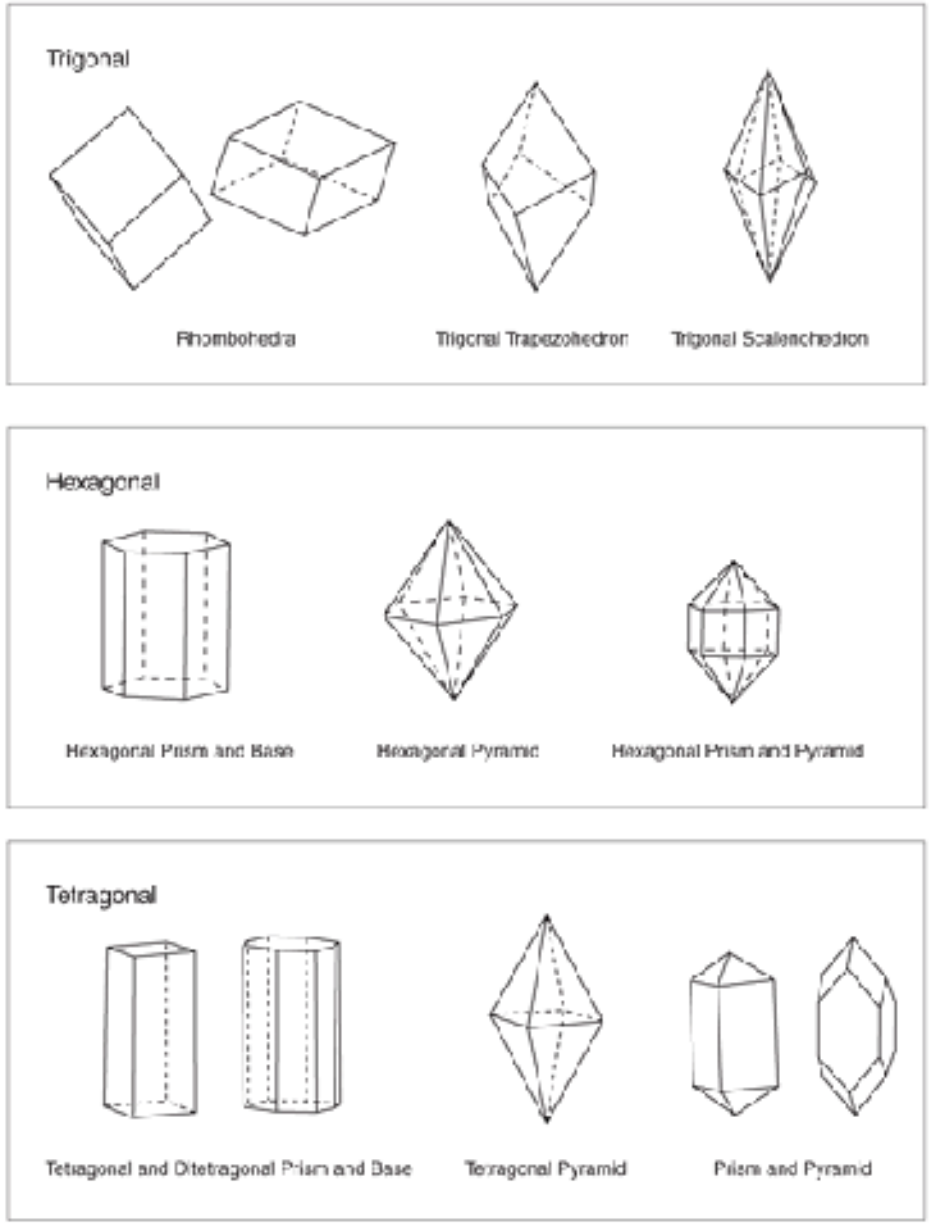
http://www.seilnacht.com/Minerale/kubisch.htm

Crystal Forms



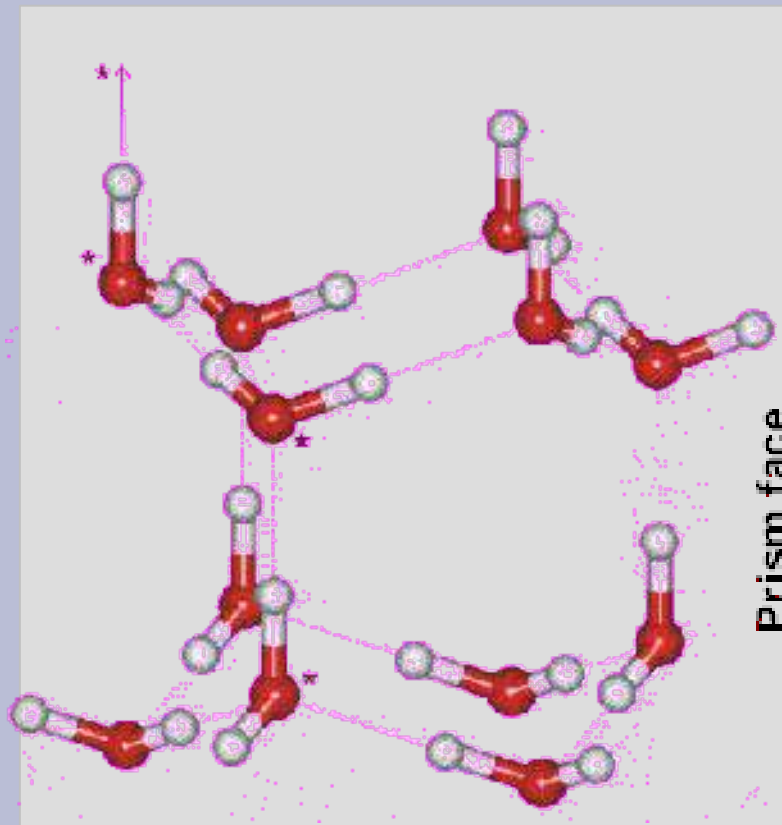


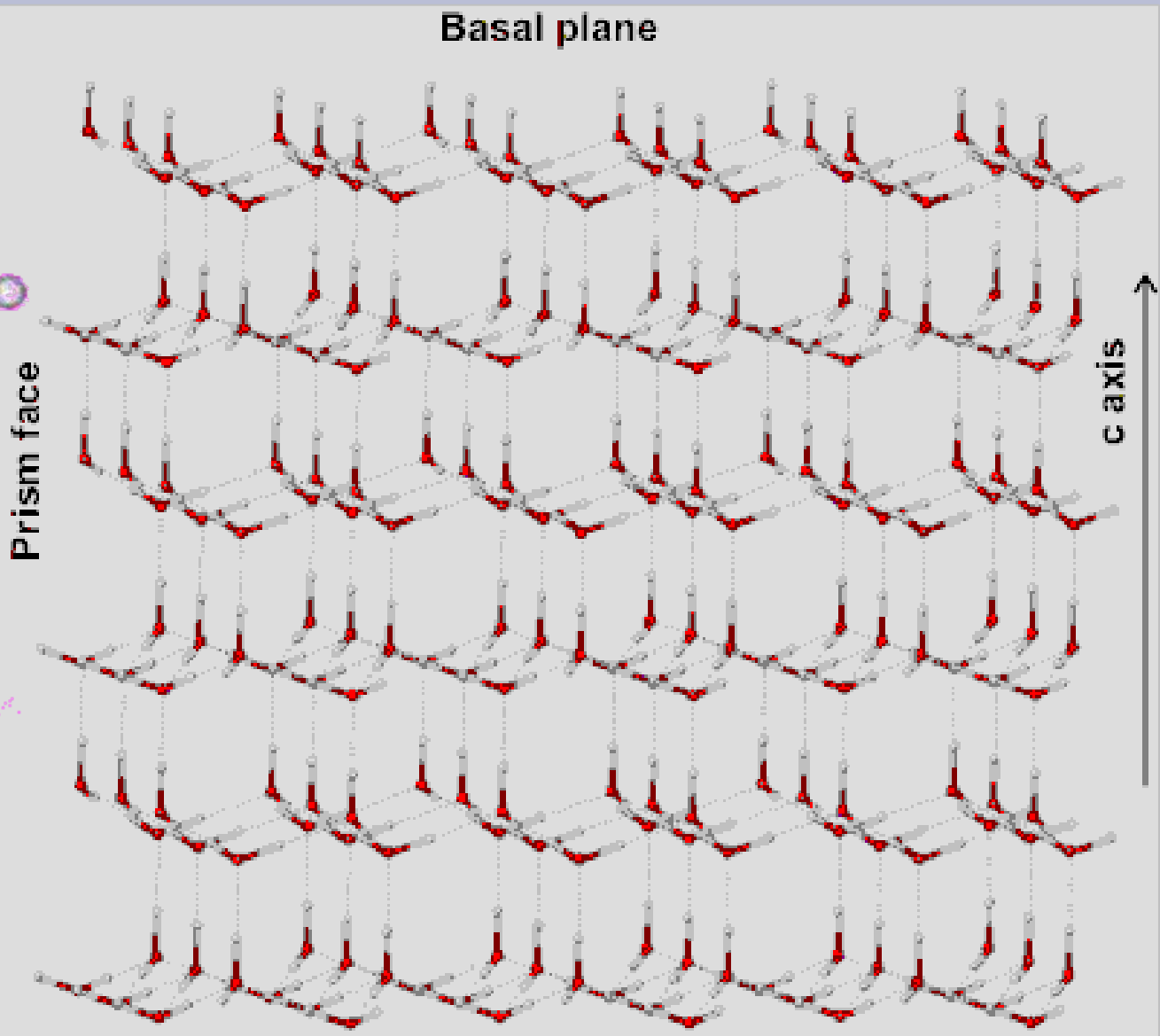




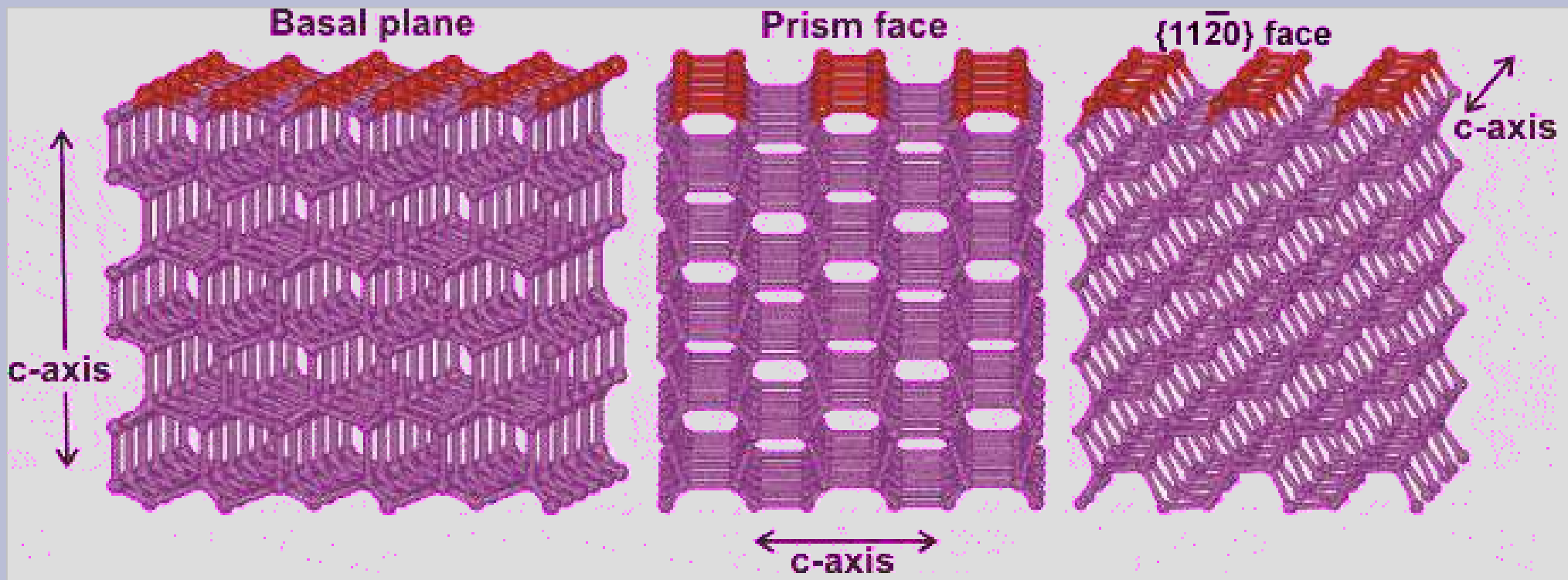
http://amonline.net.au/geoscience/minerals/crystallography.htm

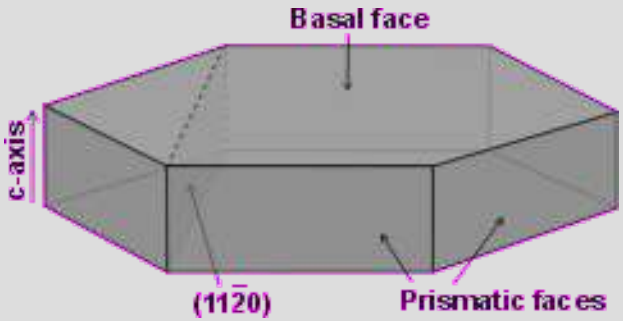
Water Ice (hexagonal Ice I_h)





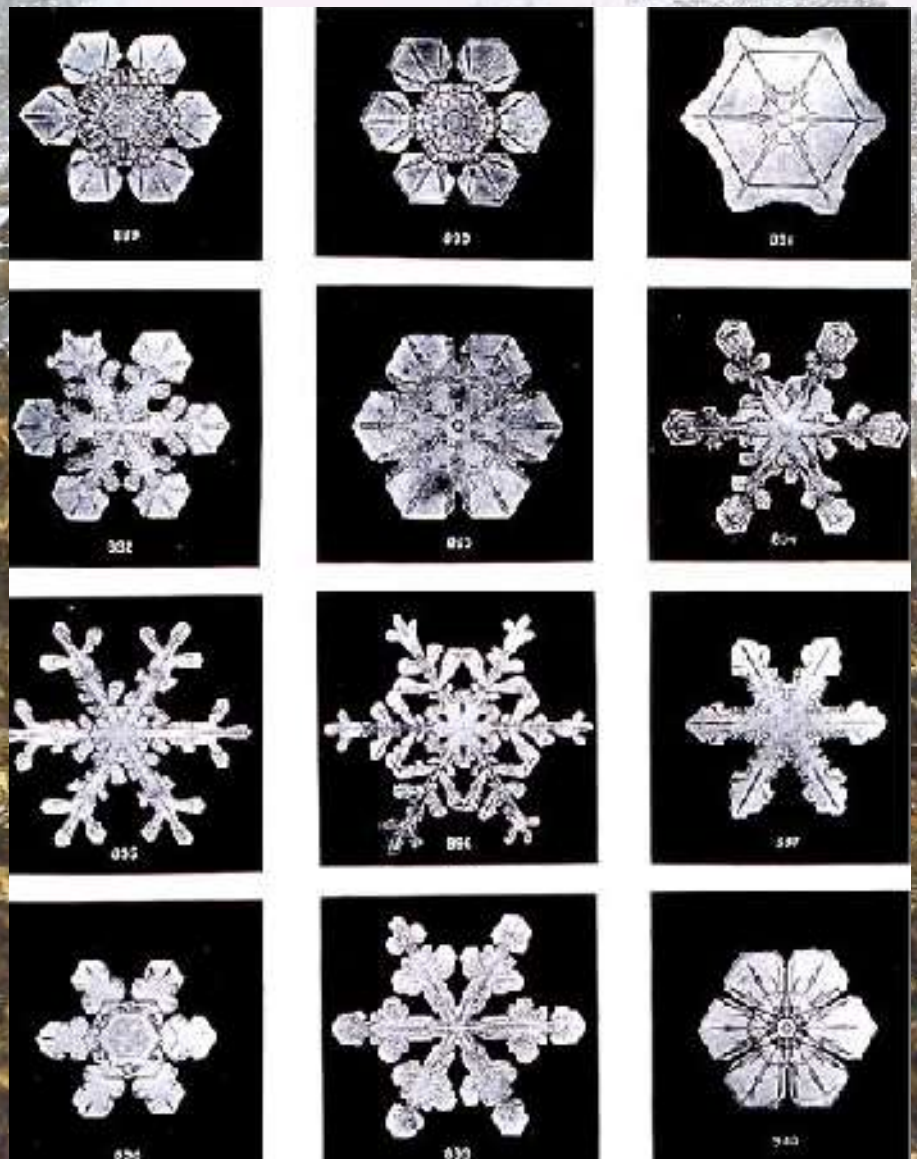
Ice Ih


Basal plane
Prism face
{11$\bar{2}$0} face
c-axis
c-axis
c-axis

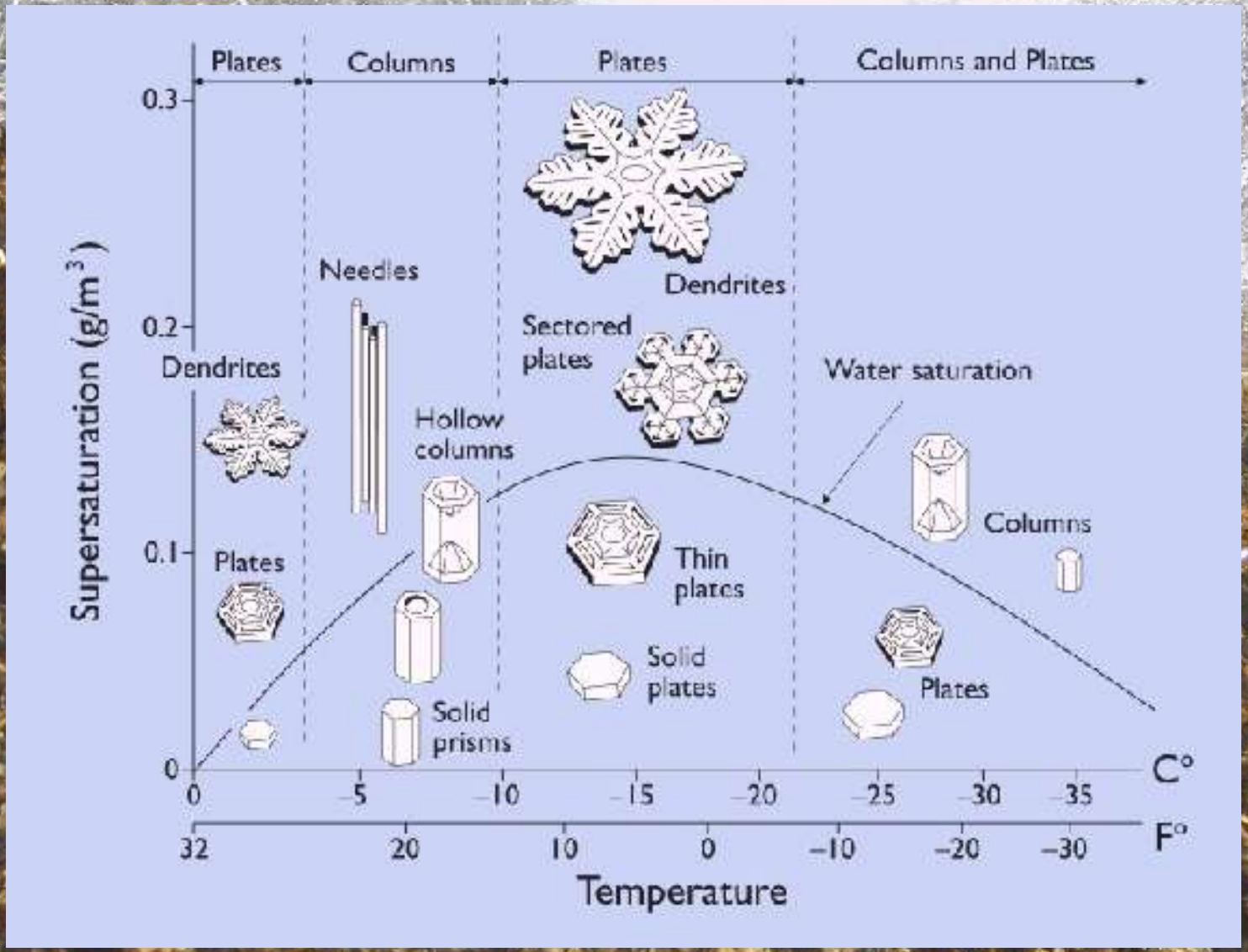

Basal face
c-axis
(11$\bar{2}$0)
Prismatic faces

Snowflakes

Wilson Bentley (1865-1931)



Snowflake morphology (Nakaya diagram)



Snowflake morphology depends on growth conditions

- temperature
- water vapour

Diffusion-limited aggregation (DLA)

- particles undergoing random walk (diffusion, Brownian motion)
- aggregate
- no reorganization
- produce “Brownian trees”


copper grown in an electrodeposition cell
from copper sulfate solution

Other fingering mechanisms

- Basic idea: Growth at the tip is easier than at the base
- A lot of different examples and names:
  - Viscuous fingering; flow in porous medium (Saffman-Taylor instability)
  - Fingering in solidification (Mullins-Sekerka instability)

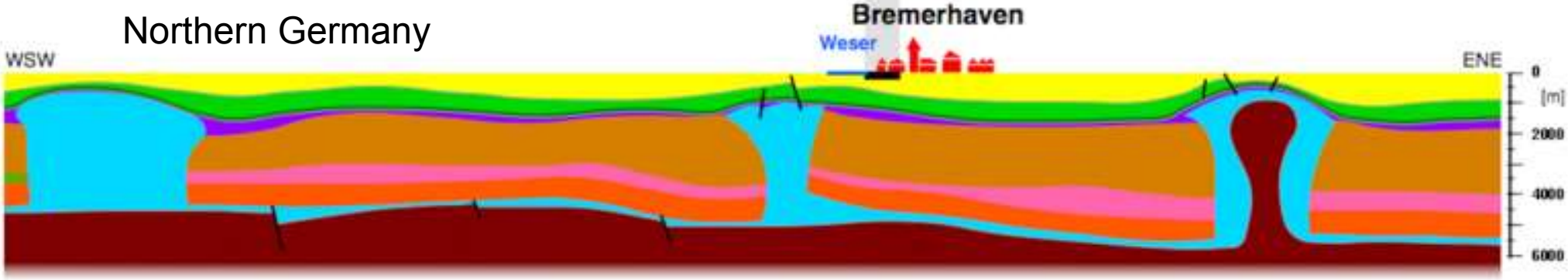

6
5
4
3
2
1
0

http://en.wikipedia.org/wiki/Salt_dome

Zagros Mountains, Iraq/Iran

Salt domes

Northern Germany




Phyllotaxis

Pineapple science

- How many spiral arms do you count?

34 and 55 spirals

Fibonacci sequence

- 1,1,3,5,8,13,21,34,55, 89,144, ...
- Each number is the sum of the two predecessors

Leonardo of Pisa (c. 1170 – c. 1250), also known as Leonardo Pisano, Leonardo Bonacci, Leonardo Fibonacci

Fibonacci sequence and Golden Ratio

Fibonacci: 1,1,3,5,8,13,21,34,55,89,144,...

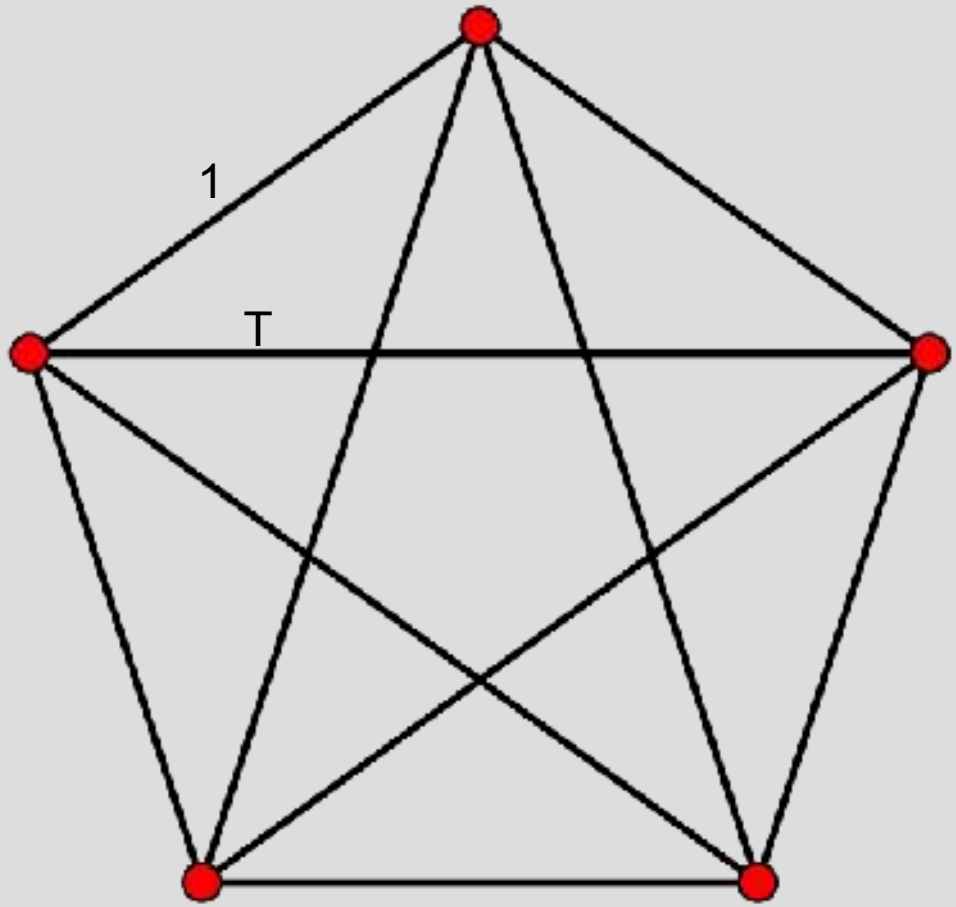
3 / 1 = 3
5 / 3 = 1.66666...
8 / 5 = 1.6
13 / 8 = 1.625
21 / 13 = 1,61538461538...
34 / 21 = 1,61904761905...
55 / 34 = 1,61764705882...
89 / 55 = 1,61818181818...

$\lim(f_n/f_{n-1}) = 1.61803398... = (1+\sqrt{5})/2$

Golden Ratio

$T = \lim(f_n/f_{n-1}) = 1.61803398... = (1+\sqrt{5})/2$

T : 1 = (T +1) : T

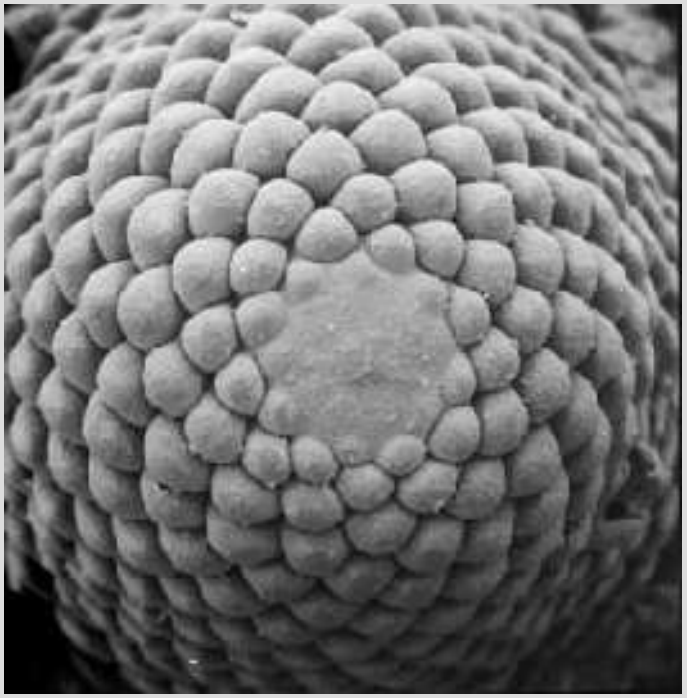


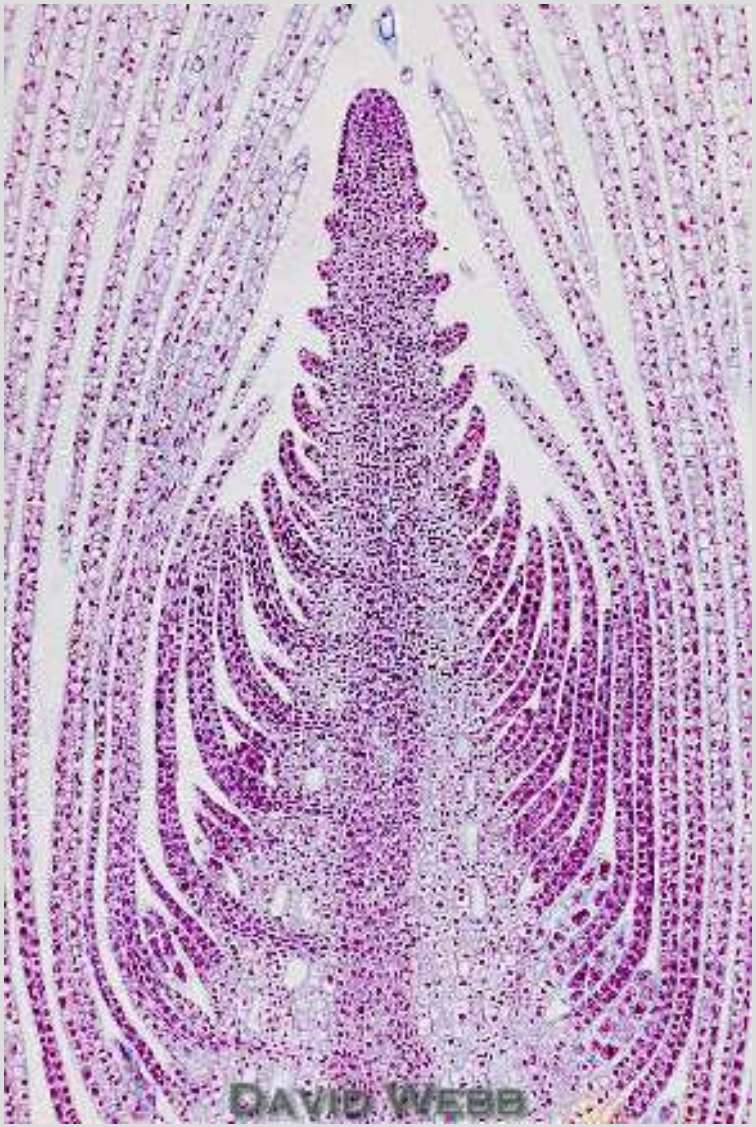
Golden angle:
360°/T ~ 222.5...°

Or, more common,
make it smaller than 180°:
360° - 360°/T ~ 137.5...°

Phyllotaxis or phyllotaxy (arrangement of leaves)

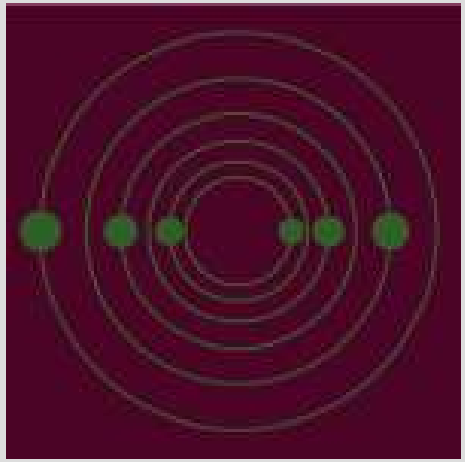
- plants: modular organisms
- tip (apex) growth:
  new modules formed on meristematic ring





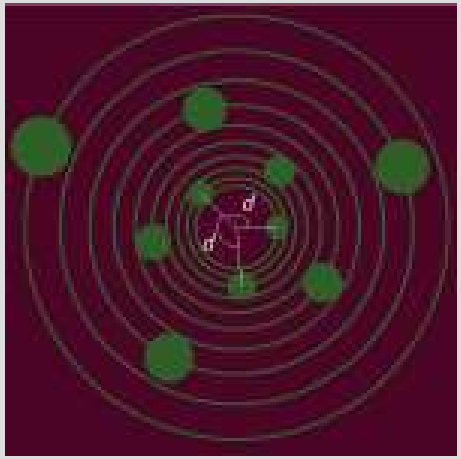
Classification:
Main types of phyllotaxis

distichous

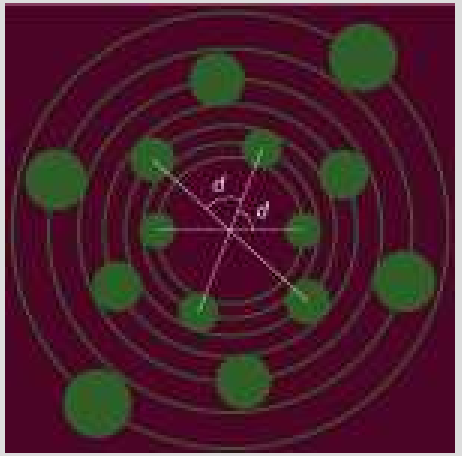


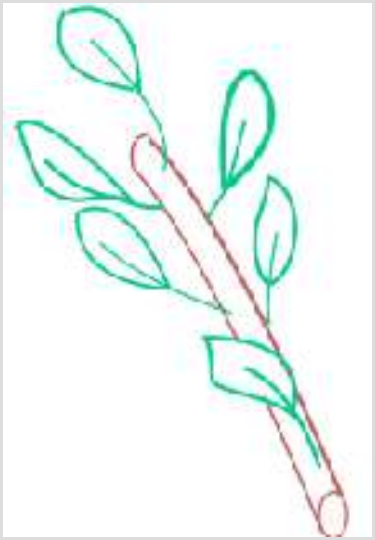
whorled

spiral



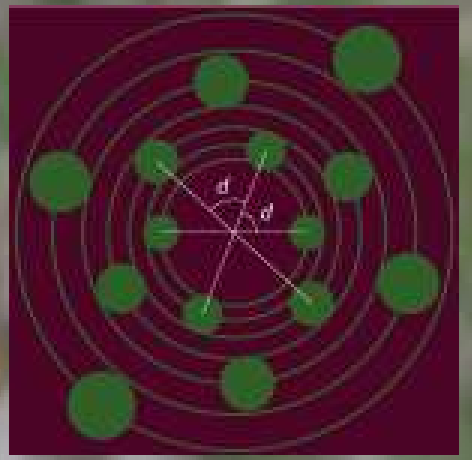
multijuguous





distichous

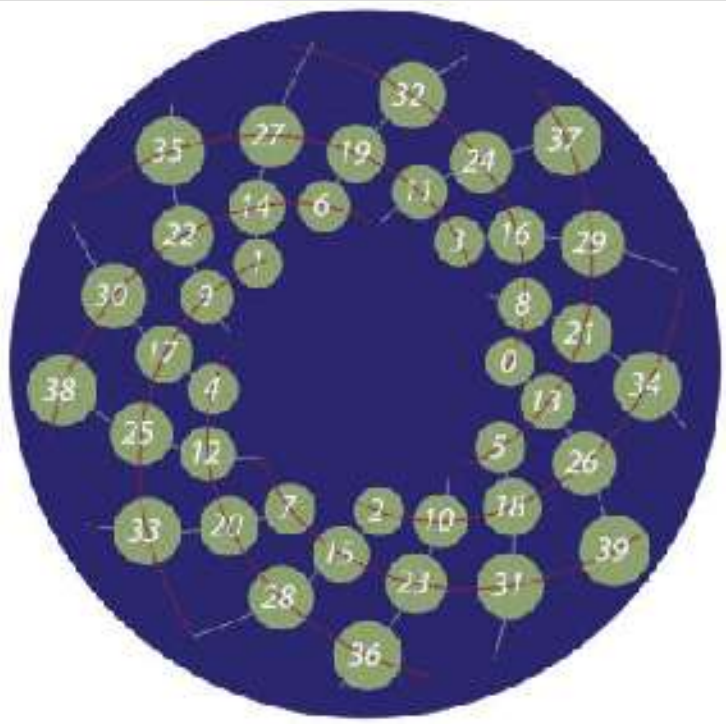
multijuguous

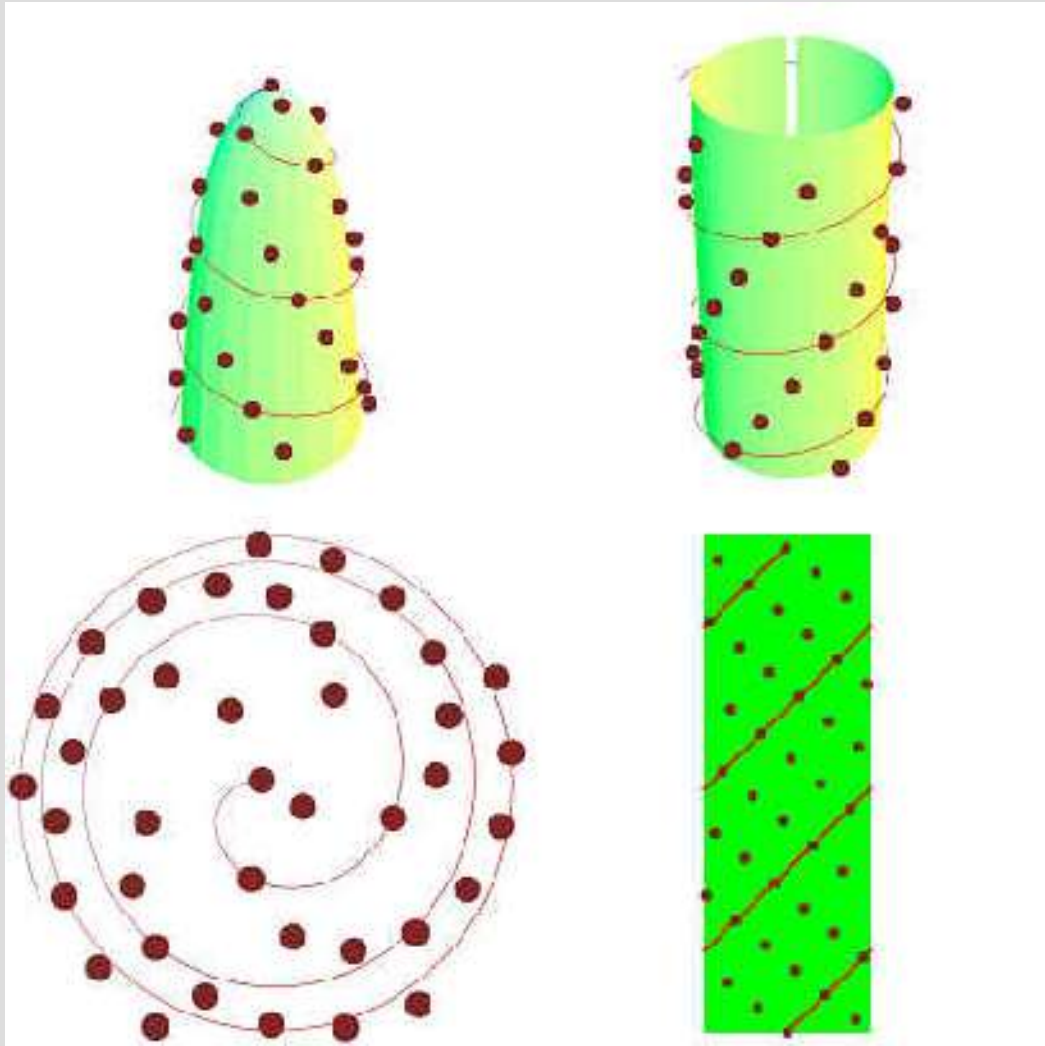



Spiral

Two spirals...

- paristiche: spirals made up of next neighbours
- **generative spiral**

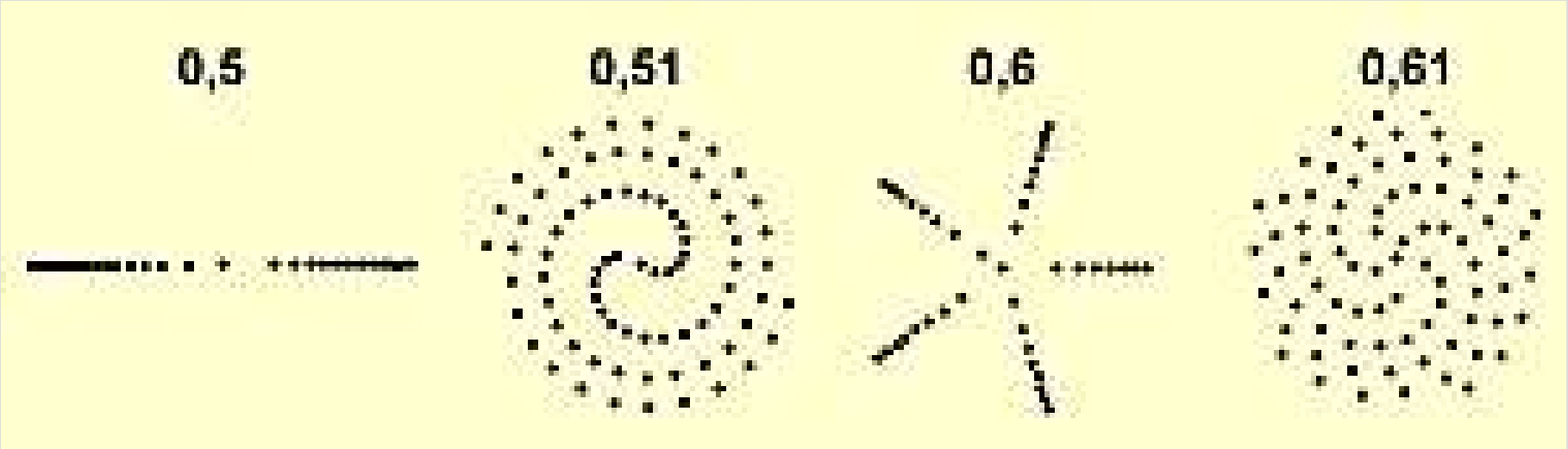




Why the golden angle?

Try different angles with the Spiral lattices applet:
http://www.math.smith.edu/phyllo//Applets/Spiral/Spiral.html

Golden angle gives the densest arrangement with least overlap



The most irrational number

There is a „most irrational“ number, and it turns out (surprise, surprise) the golden number.

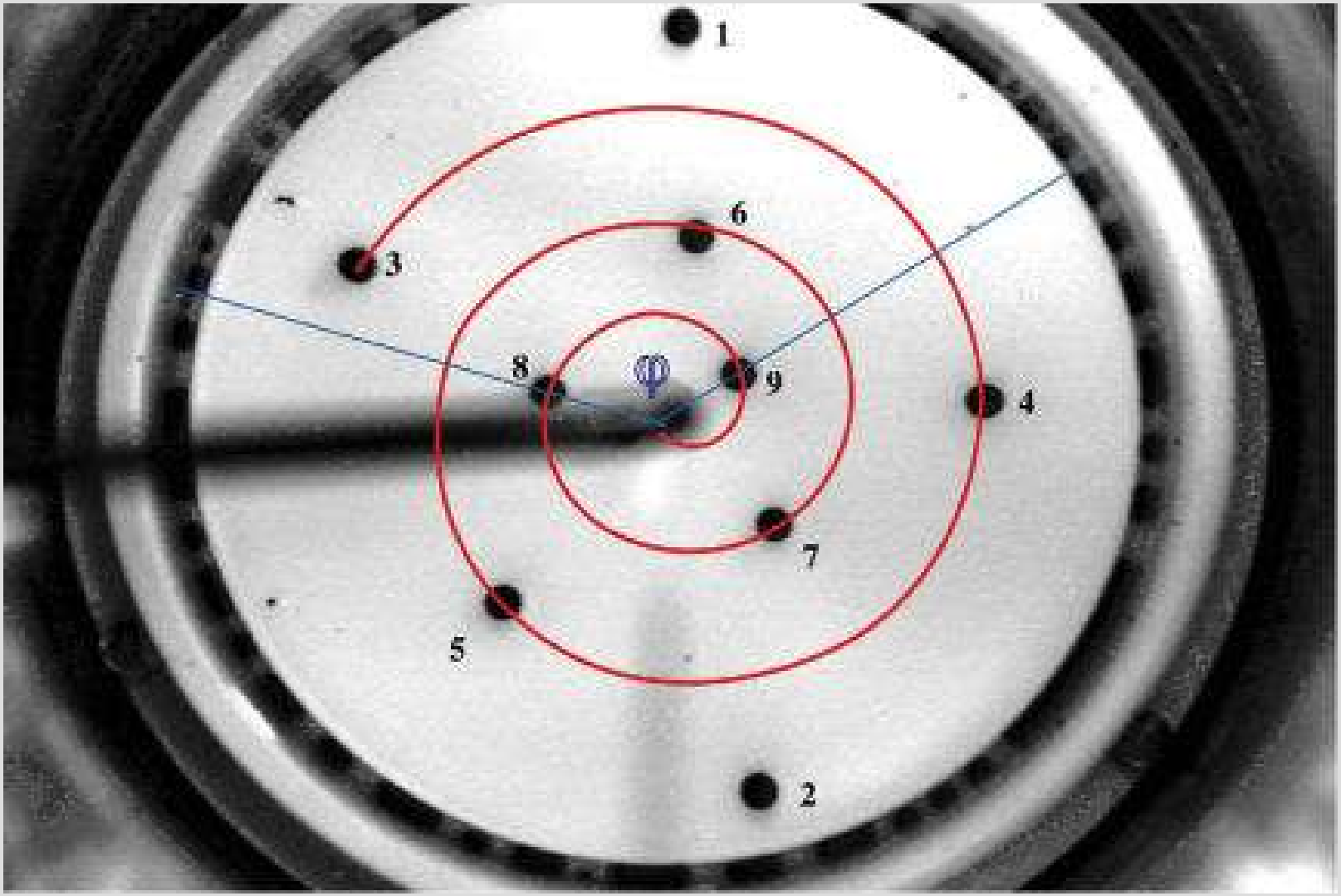
...

It is the most badly „approximable-by-rational“ number there is!

Its „badness“ is exceeded only by the awkwardness of the preceding sentence.

(Adam, Mathematics in Nature, p220)

An Experiment: Phyllotaxis as self-organization



Douady and Couder 1996

Patterns in Nature
Outline

1. Introduction
2. Waves and oscillations
3. Regularity and chaos
4. Animal cooperation
5. Spatial patterns
6. Aggregation and growth processes
7. Cellular automata
8. Fractals
9. Miscellaneous topics
10. Concluding session