

A close-up photograph of a frozen stream bed. The surface is covered in numerous sharp, translucent ice shards of varying sizes, some of which are embedded in a layer of white snow or frost. Below the ice, small, light-colored pebbles and stones are visible through the clear, frozen water. The overall texture is rough and intricate.

Patterns in Nature 6

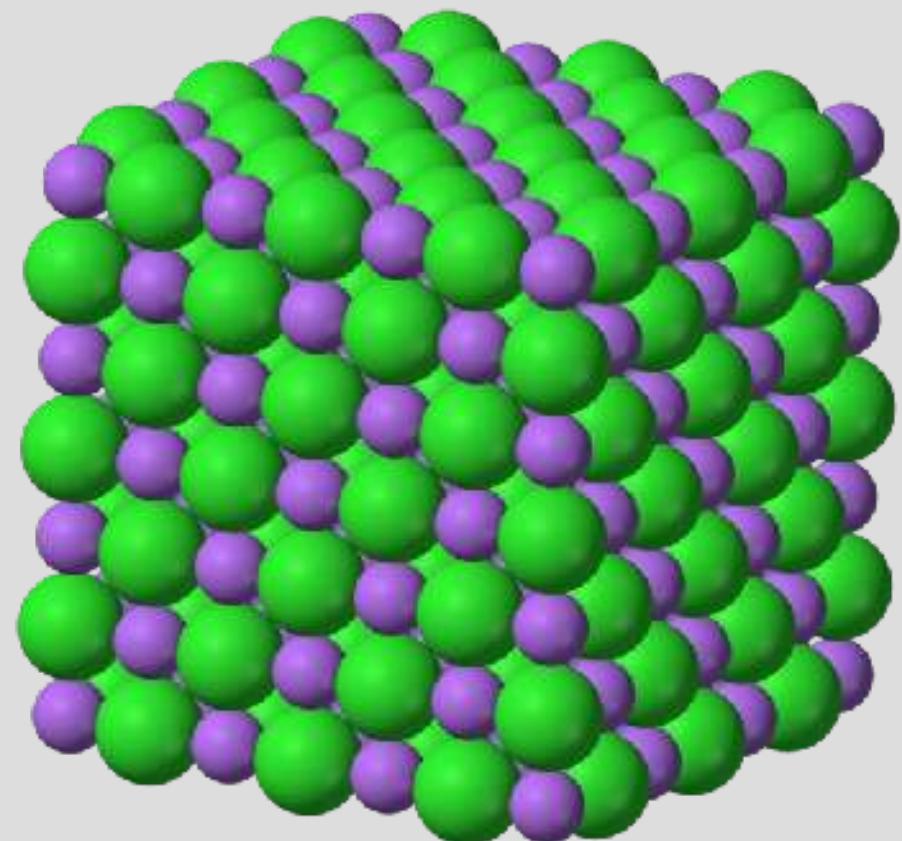
Growth processes

Stephan Matthiesen

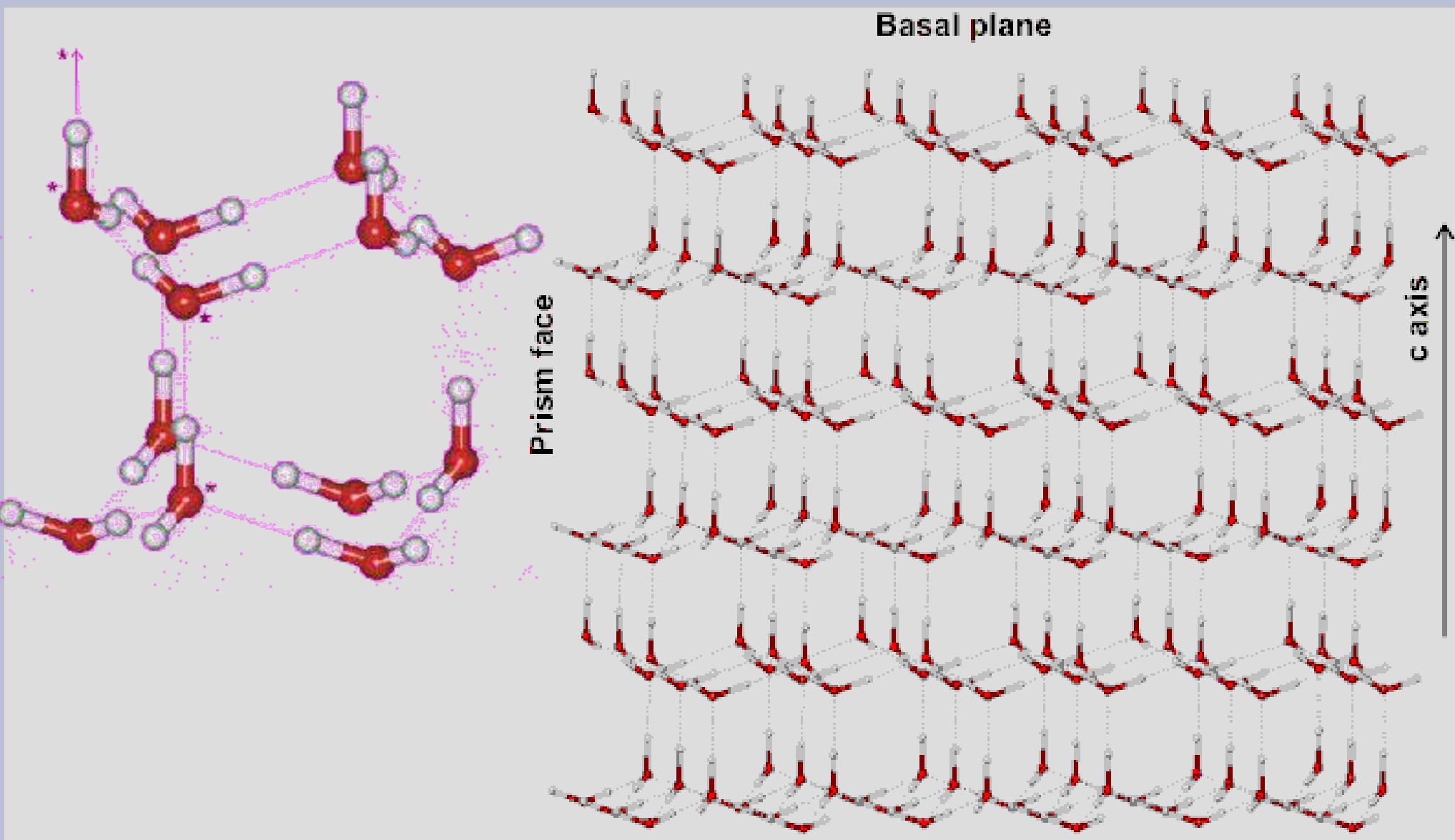
Crystals



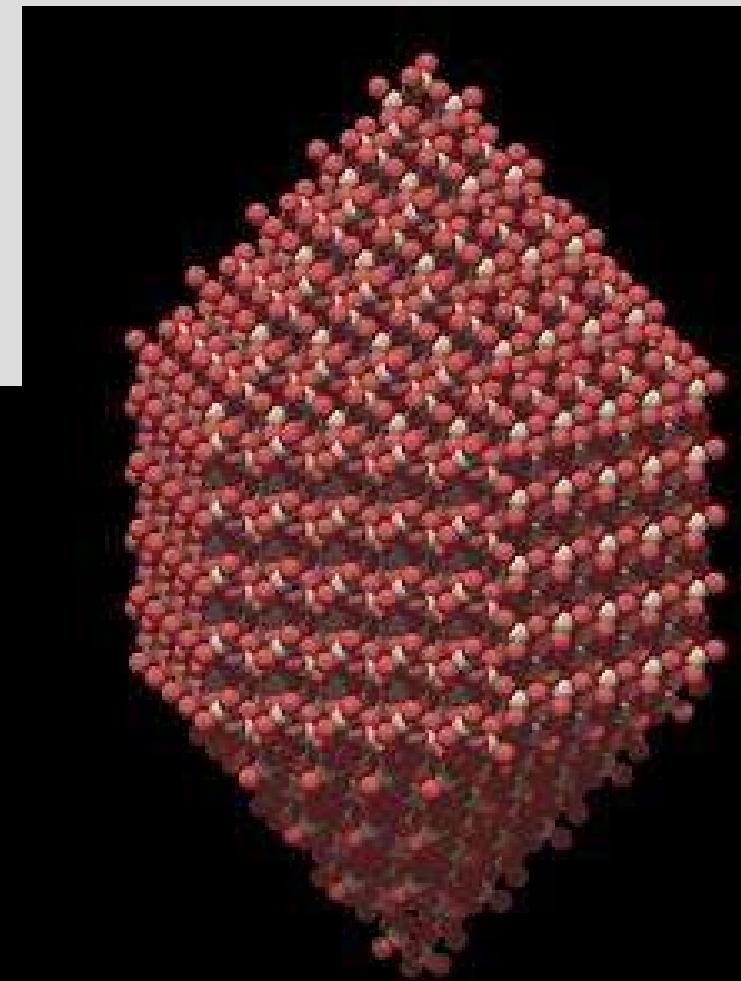
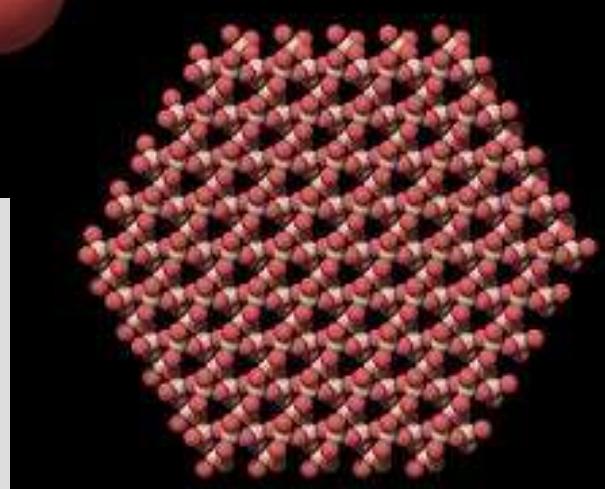
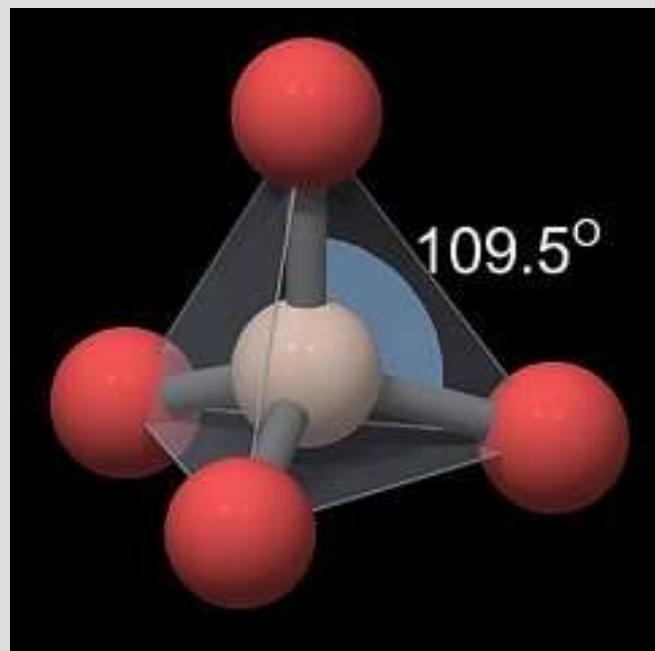
Salt (Sodium Chloride)



Water Ice (hexagonal Ice I_h)

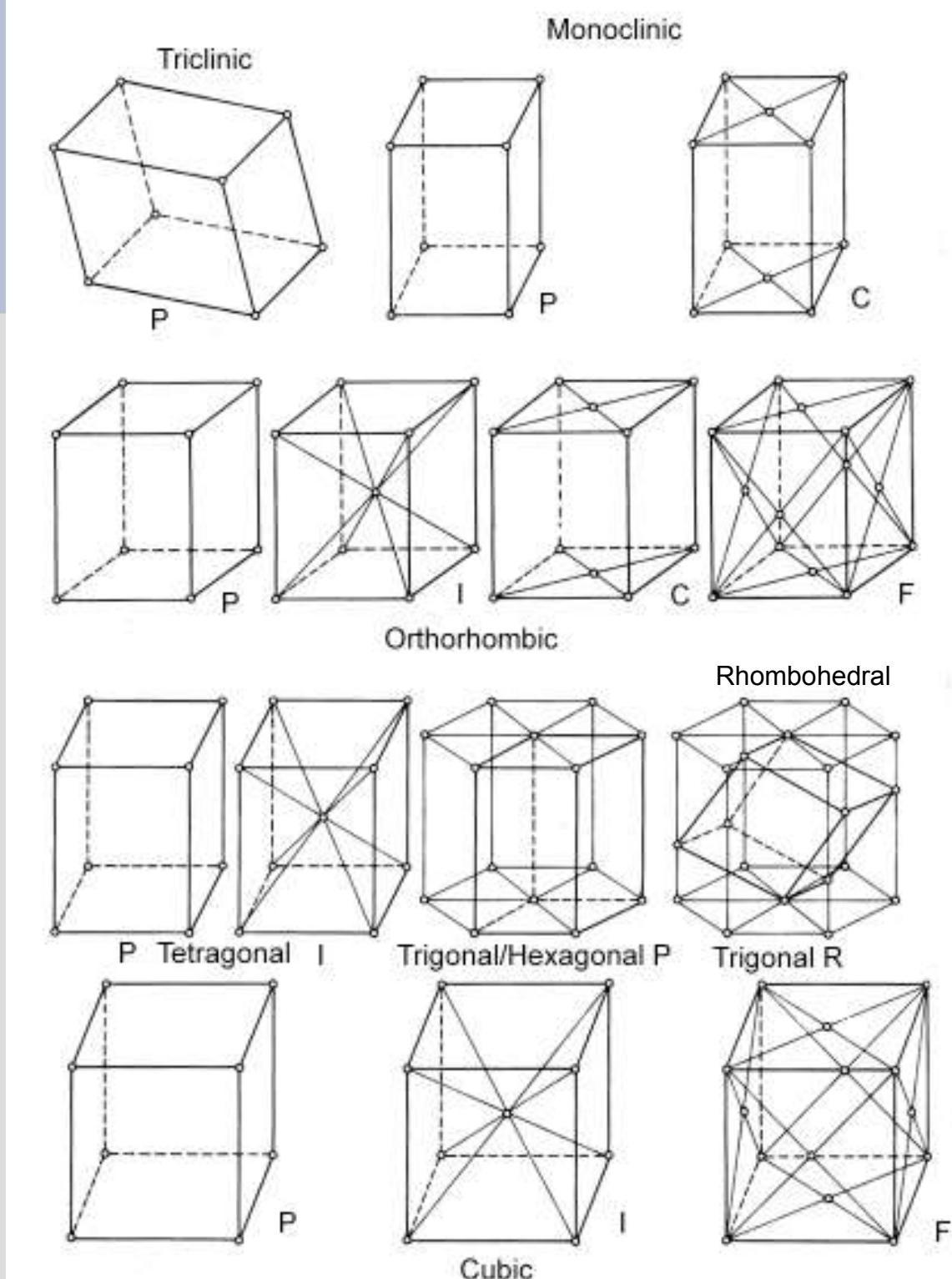


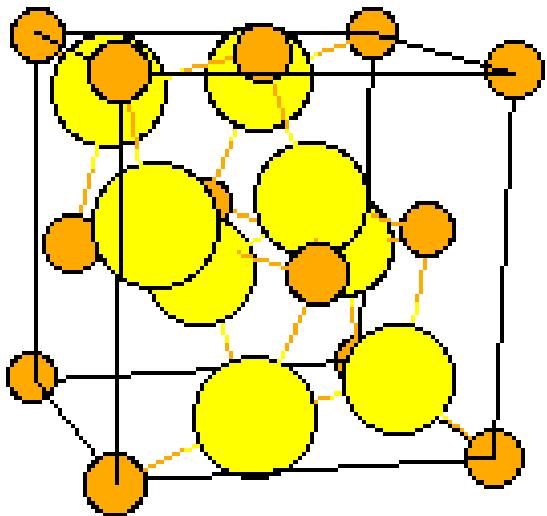
Quartz (SiO_2)



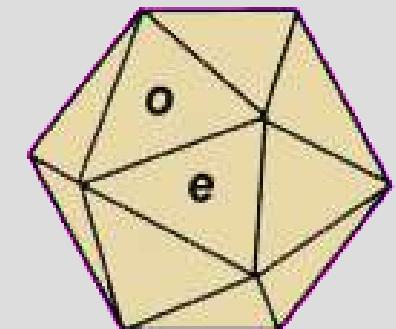
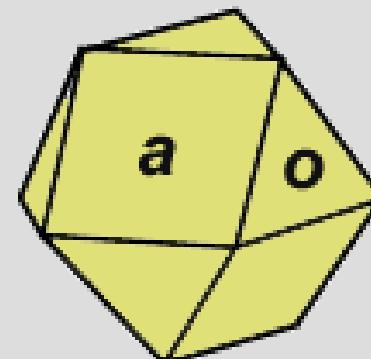
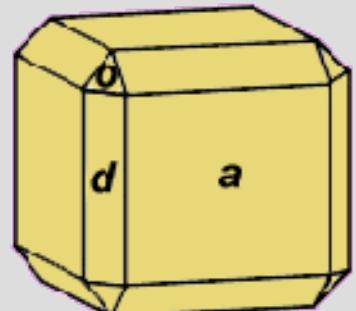
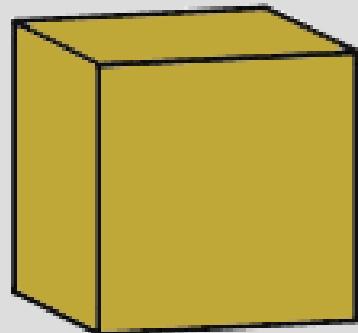
Crystal Systems

- 7 Crystal Systems
- 14 Bravais lattices
- 230 space groups





Crystal Forms of Pyrite (cubic)



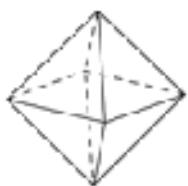
Crystal Forms

Cubic

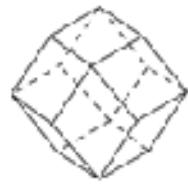


Cube

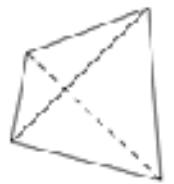
Octahedron



Dodecahedron



Tetrahedron



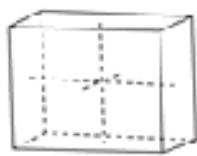
Pyritohedron



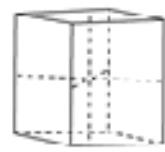
Cube & Pyritohedron



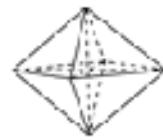
Orthorhombic



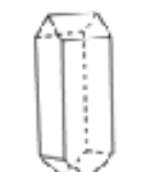
Pinacoids



Prism and Basal Pinacoid



Pyramidal

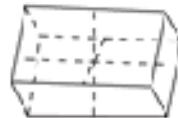


Orthorhombic Dipyramidal and Prism



Prism, Domes and Two Pinacoids

Monoclinic



Domes and Pinacoid



Prism and Pinacoid



Trigonal



Rhombohedra

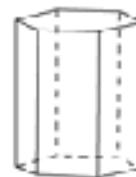


Trigonal Trapezohedron



Trigonal Scalenohedron

Hexagonal



Hexagonal Prism and Base

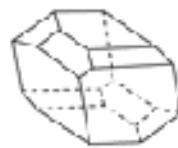


Hexagonal Pyramid

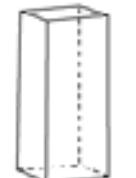


Hexagonal Prism and Pyramid

Triclinic



Tetragonal



Tetragonal and Ditetragonal Prism and Base

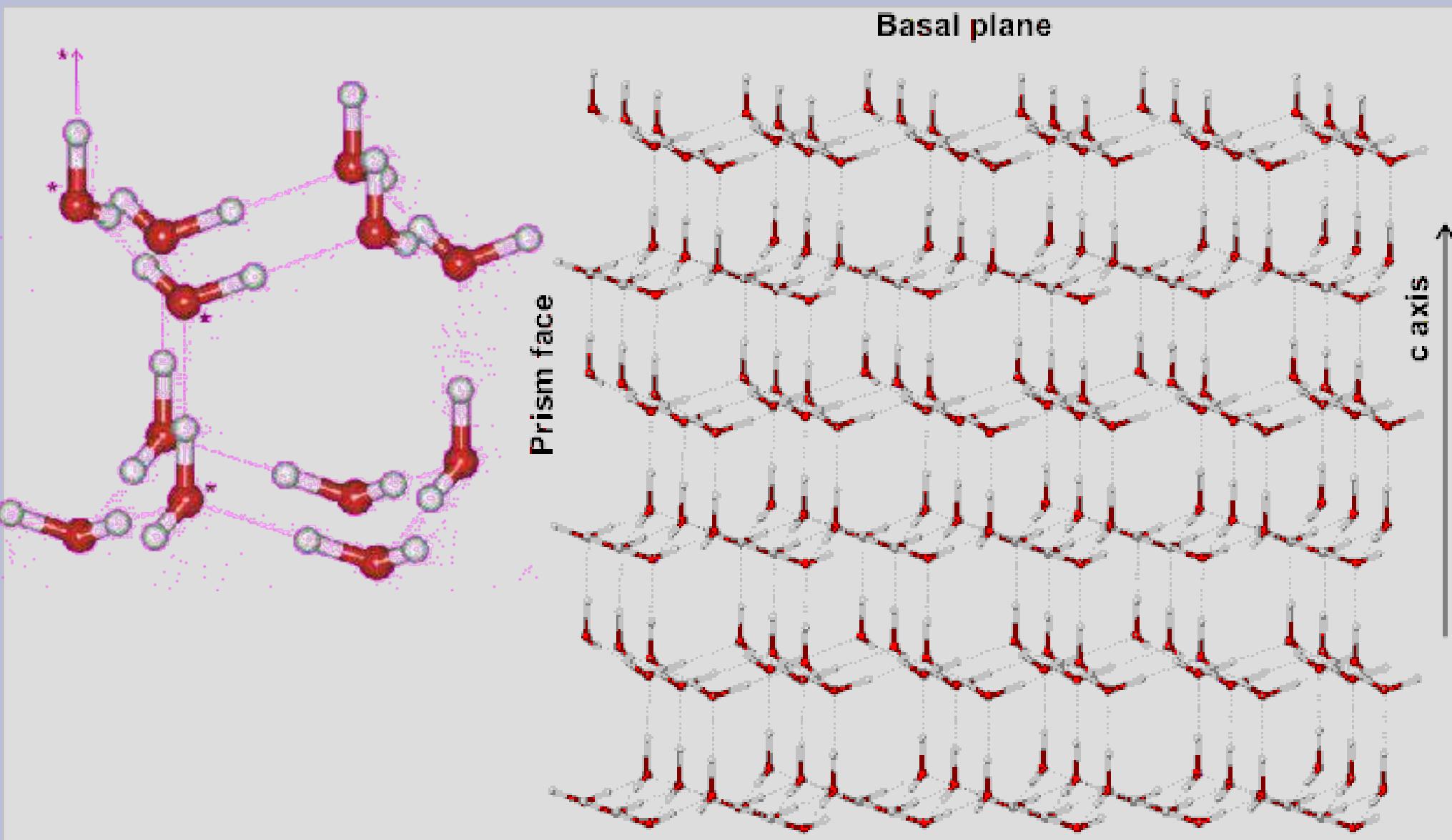


Tetragonal Pyramid

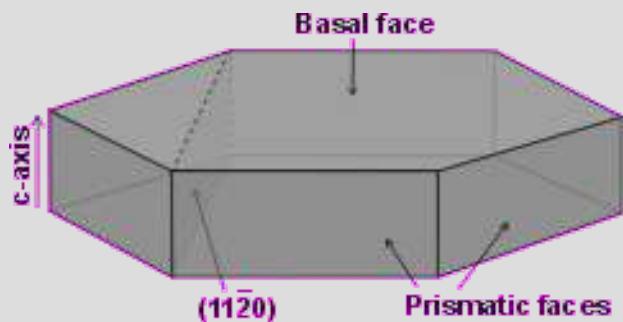
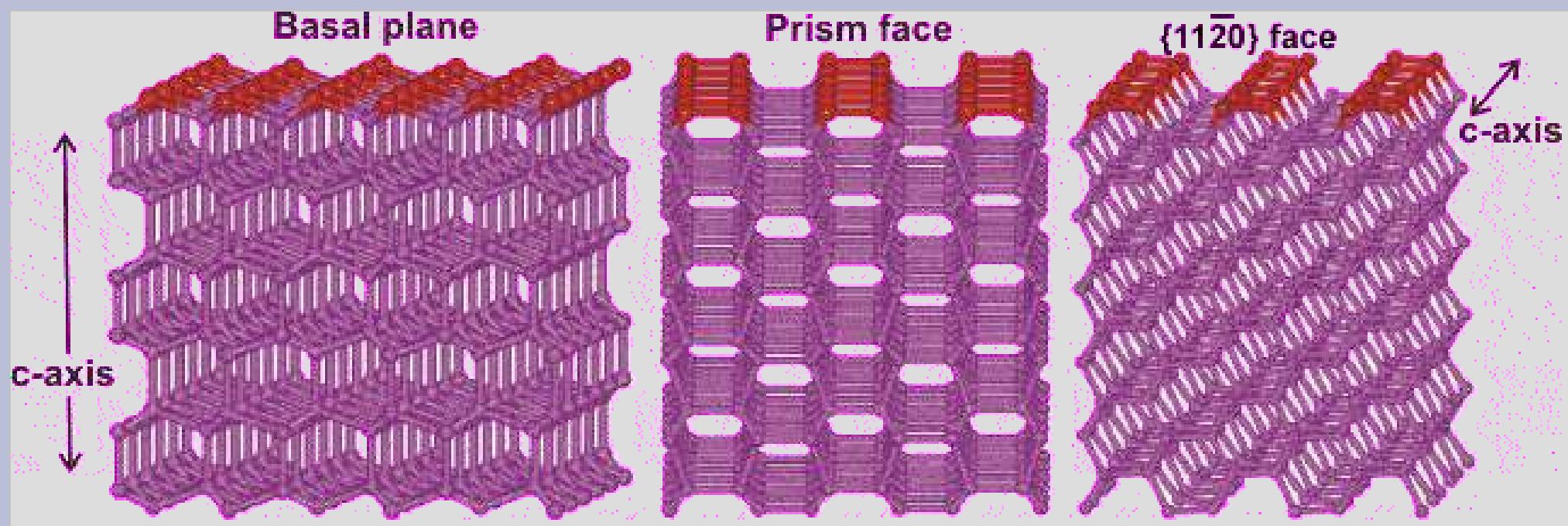


Prism and Pyramid

Water Ice (hexagonal Ice I_h)

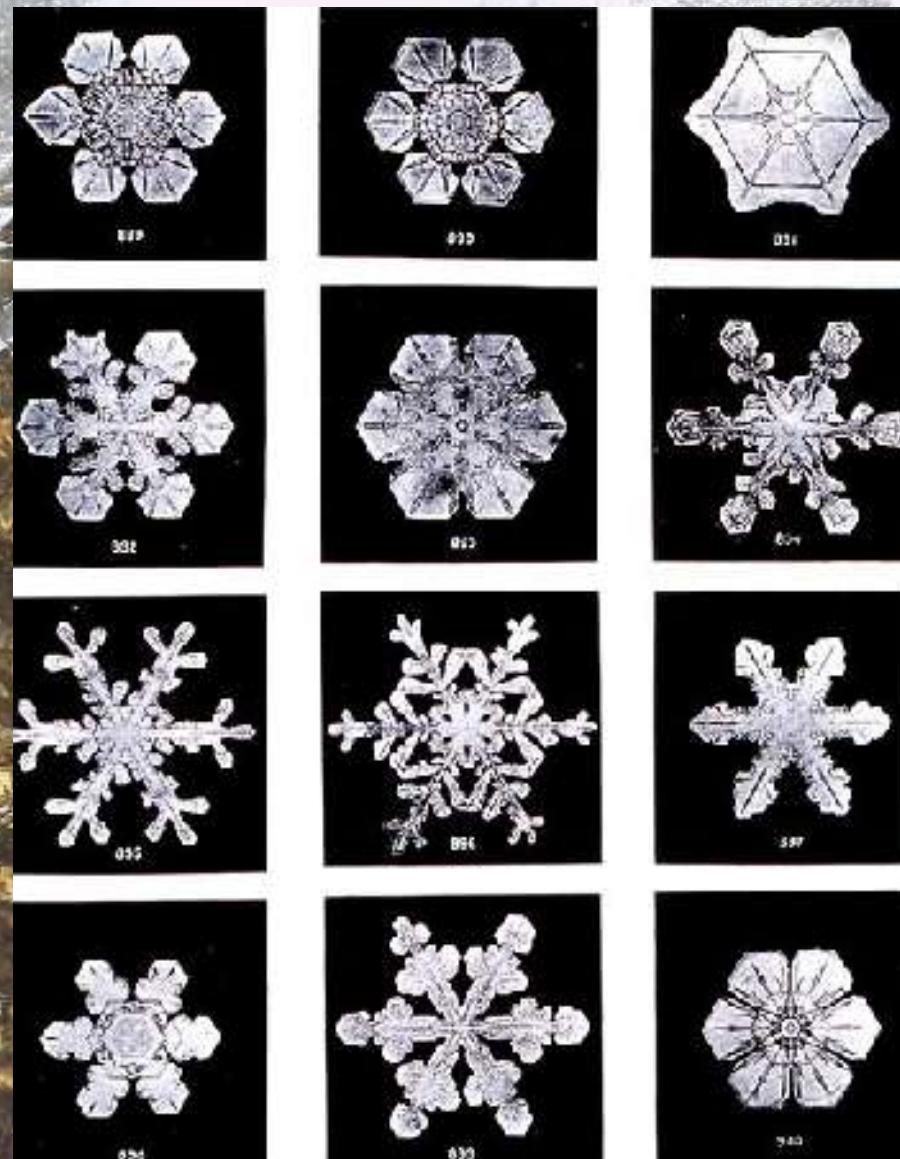


Ice Ih

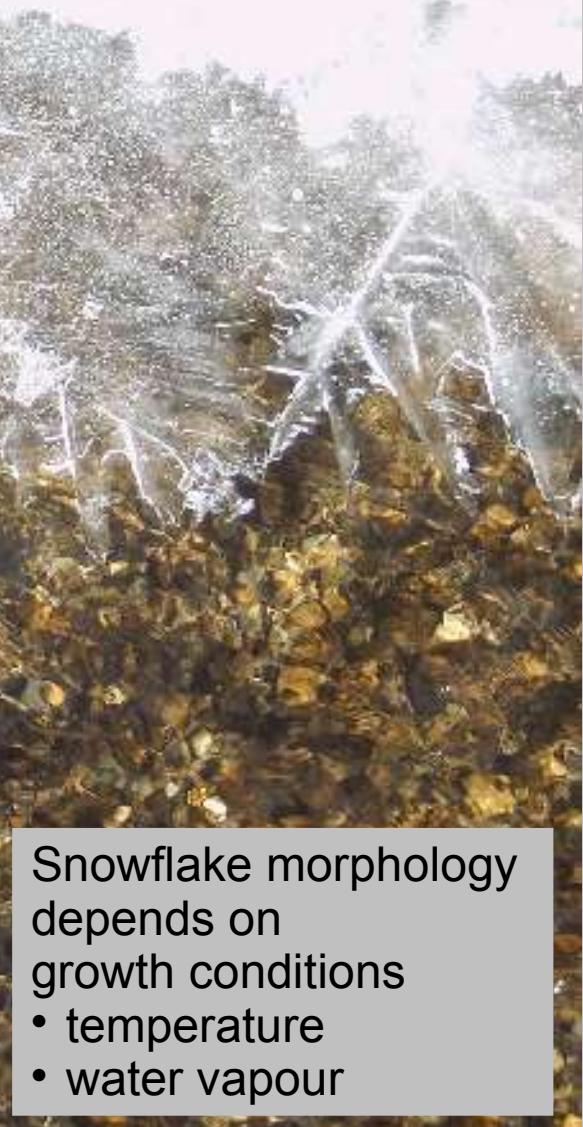
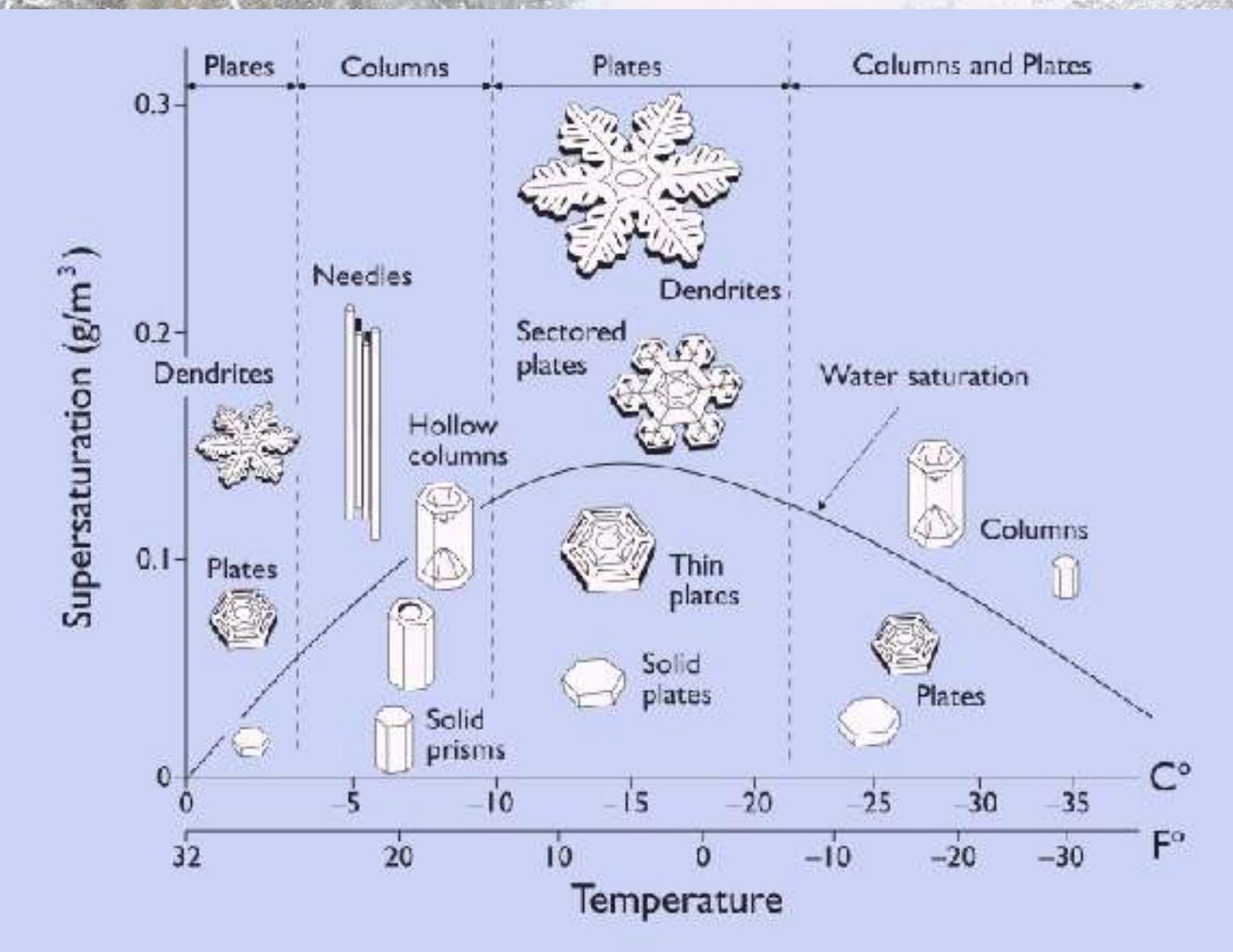


Snowflakes

Wilson Bentley (1865-1931)



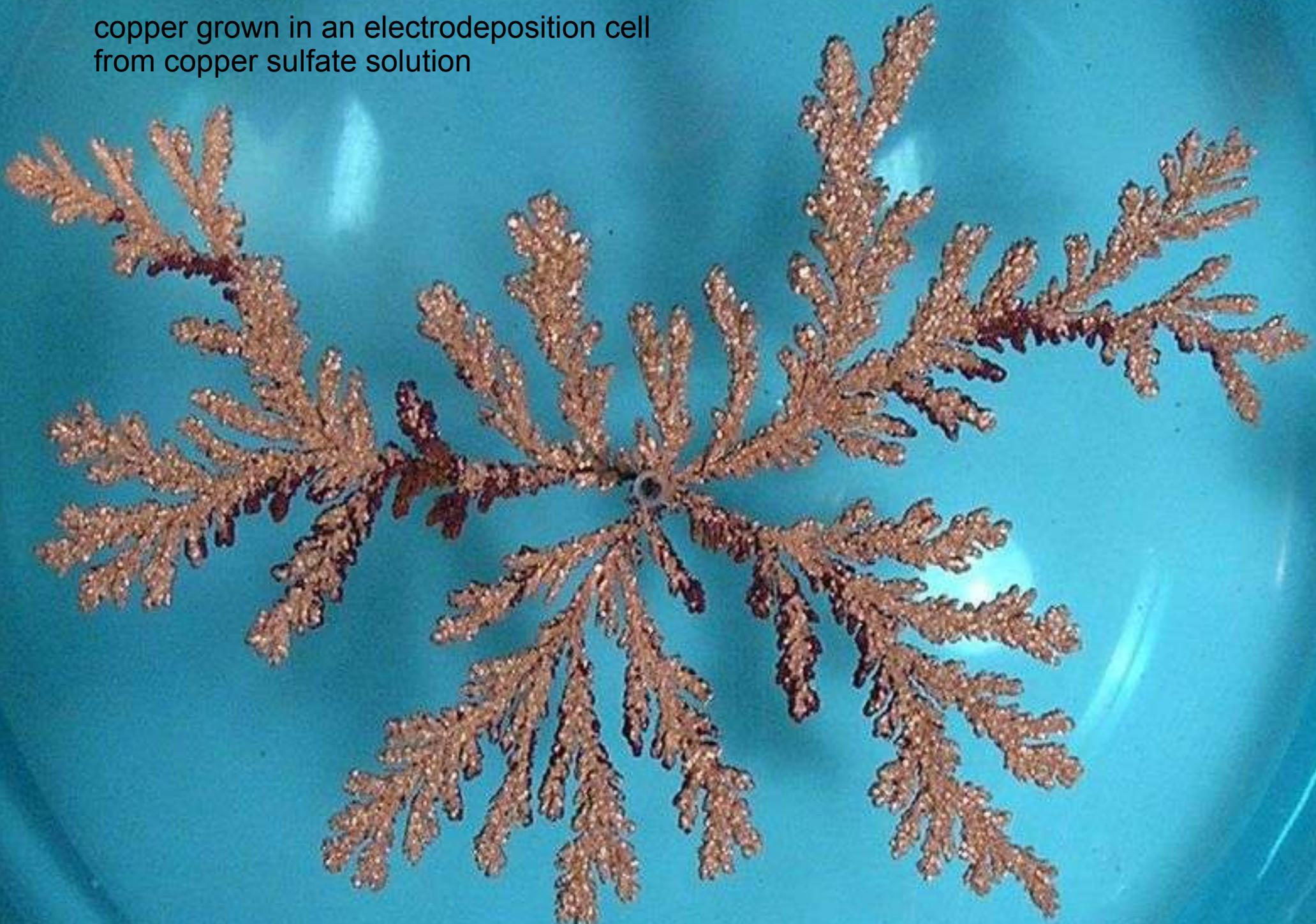
Snowflake morphology (Nakaya diagram)



Diffusion-limited aggregation (DLA)

- particles undergoing **random walk** (diffusion, Brownian motion)
- aggregate
- no reorganization
- produce “Brownian trees”

copper grown in an electrodeposition cell
from copper sulfate solution



Other fingering mechanisms

- Basic idea: Growth at the tip is easier than at the base
- A lot of different examples and names:
 - Viscuous fingering; flow in porous medium (Saffman-Taylor instability)
 - Fingering in solidification (Mullins-Sekerka instability)



6

5

= 4

3

= 2

1

< 10 <







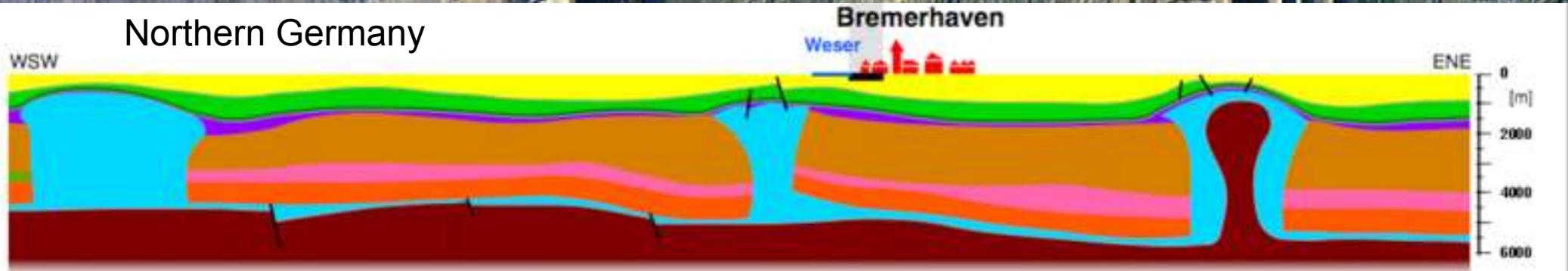
http://en.wikipedia.org/wiki/Salt_dome

Zagros Mountains, Iraq/Iran

Salt domes



Northern Germany



A close-up photograph of a green aloe vera plant. The image shows several thick, fleshy leaves with serrated edges and small, sharp spines. The leaves are arranged in a distinct whorl pattern, radiating from a central point. The lighting highlights the texture of the plant's surface.

Phyllotaxis

Pineapple science

- How many spiral arms do you count?





34 and 55 spirals

Fibonacci sequence

- 1, 1, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...
- Each number is the sum of the two predecessors



Leonardo of Pisa (c. 1170 – c. 1250), also known as Leonardo Pisano, Leonardo Bonacci, Leonardo Fibonacci

Fibonacci sequence and Golden Ratio

Fibonacci: 1,1,3,5,8,13,21,34,55,89,144,...

$$3 / 1 = 3$$

$$5 / 3 = 1.66666\dots$$

$$8 / 5 = 1.6$$

$$13 / 8 = 1.625$$

$$21 / 13 = 1,61538461538\dots$$

$$34 / 21 = 1,61904761905\dots$$

$$55 / 34 = 1,61764705882\dots$$

$$89 / 55 = 1,61818181818\dots$$

$$\lim(f_n/f_{n-1}) = 1.61803398\dots = (1+\sqrt{5})/2$$

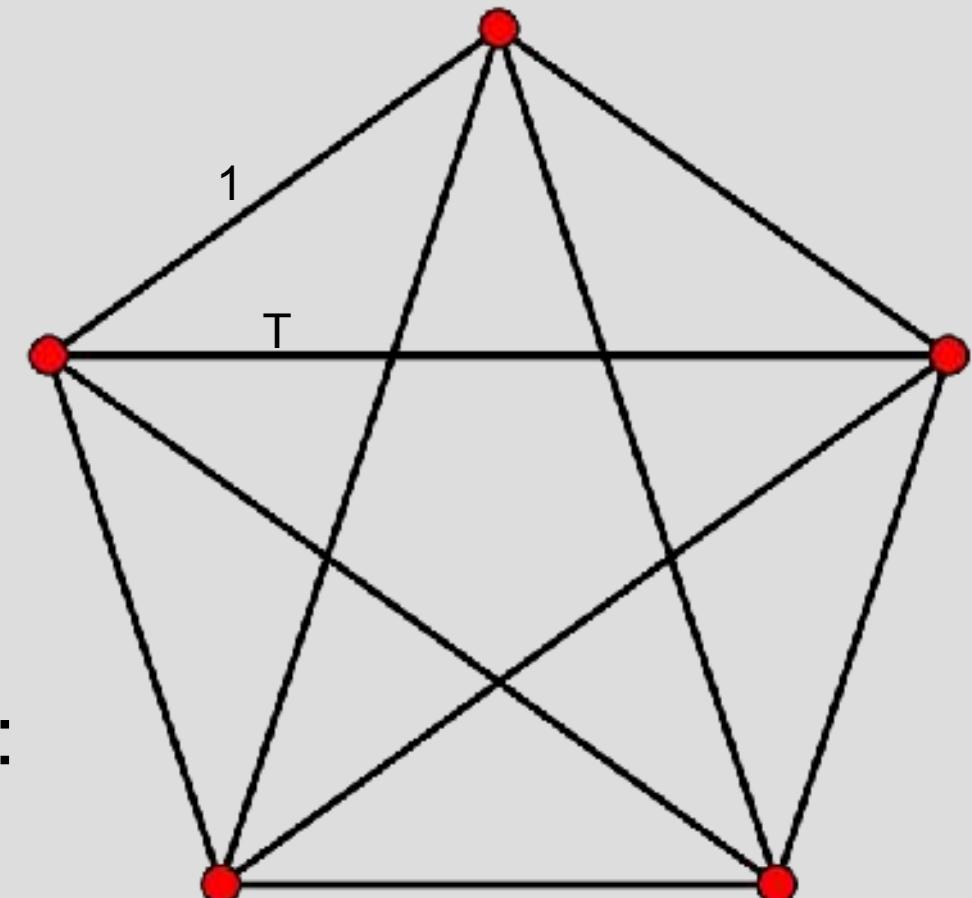
Golden Ratio

$$T = \lim(f_n/f_{n-1}) = 1.61803398\dots = (1+\sqrt{5})/2$$

$$T : 1 = (T + 1) : T$$

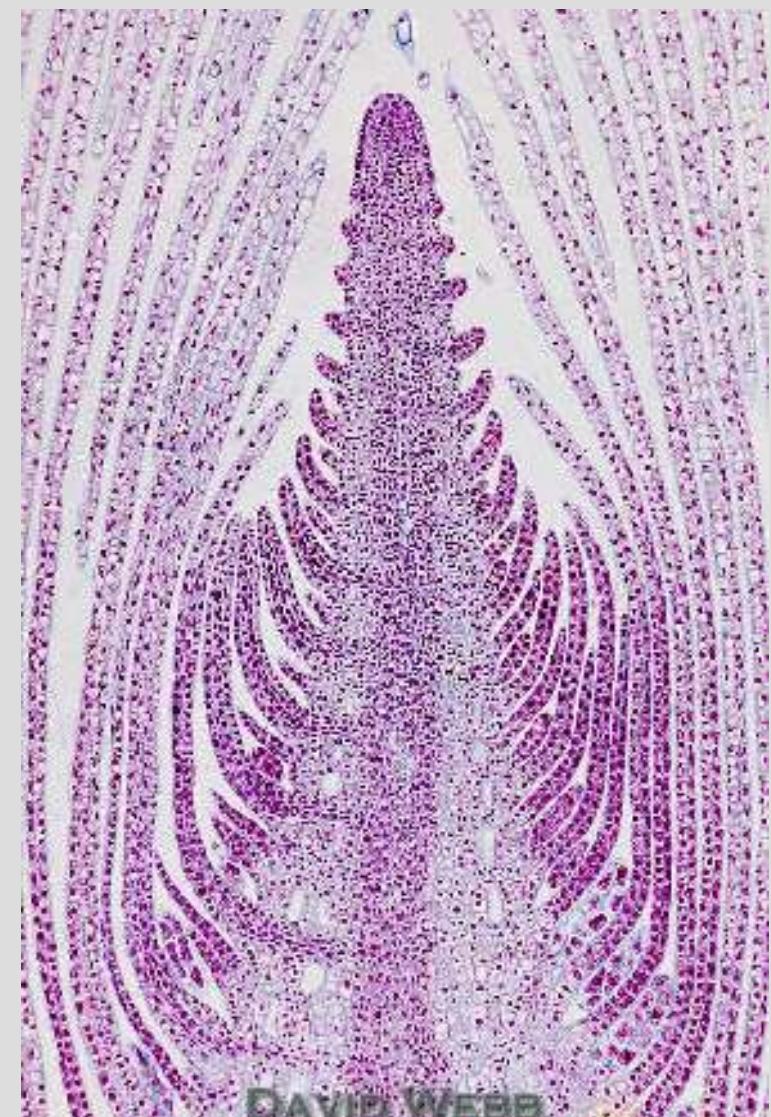
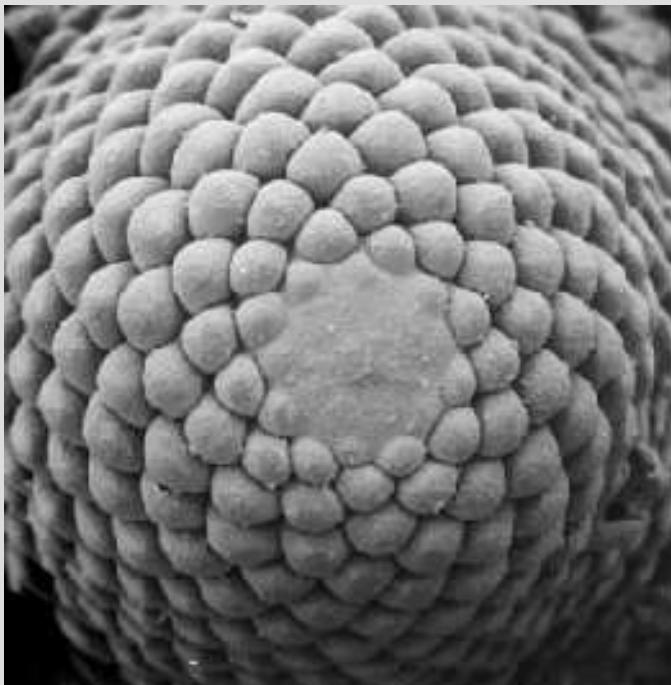
Golden angle:
 $360^\circ/T \sim 222.5\dots^\circ$

Or, more common,
make it smaller than 180° :
 $360^\circ - 360^\circ/T \sim 137.5\dots^\circ$



Phyllotaxis or phyllotaxy (arrangement of leaves)

- plants: modular organisms
- tip (apex) growth:
new modules formed on
meristematic ring



Classification: Main types of phyllotaxis

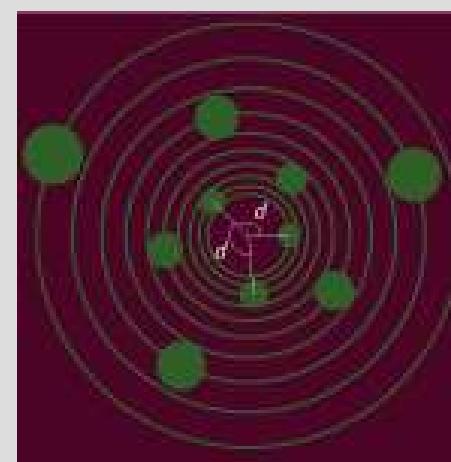
distichous



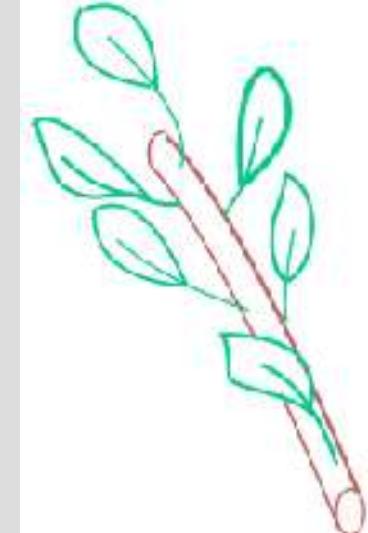
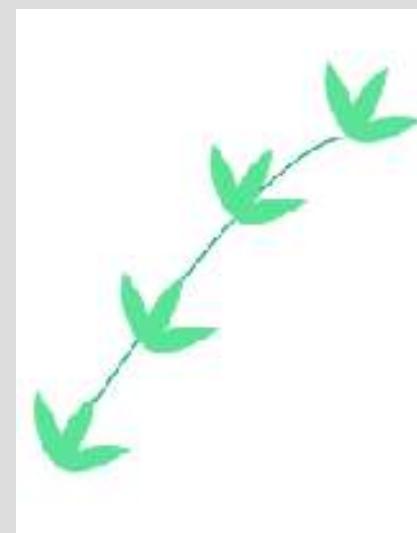
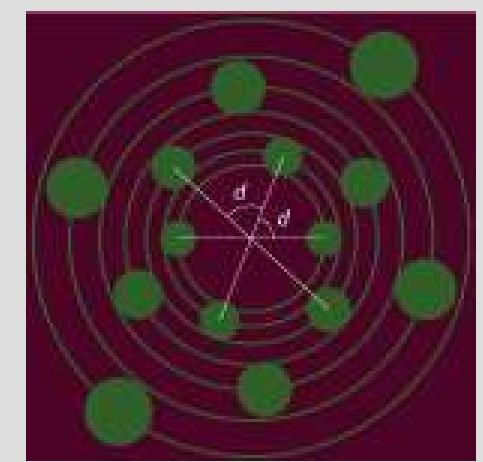
whorled



spiral



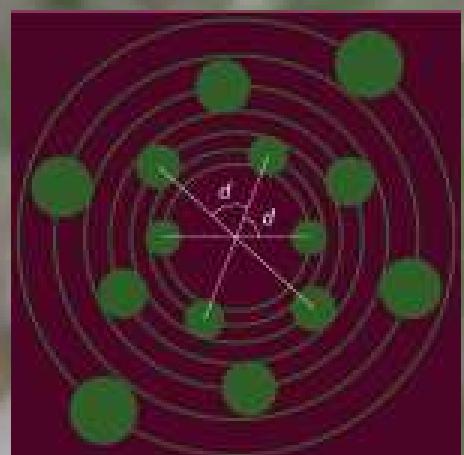
multijugous



distichous



multijugous

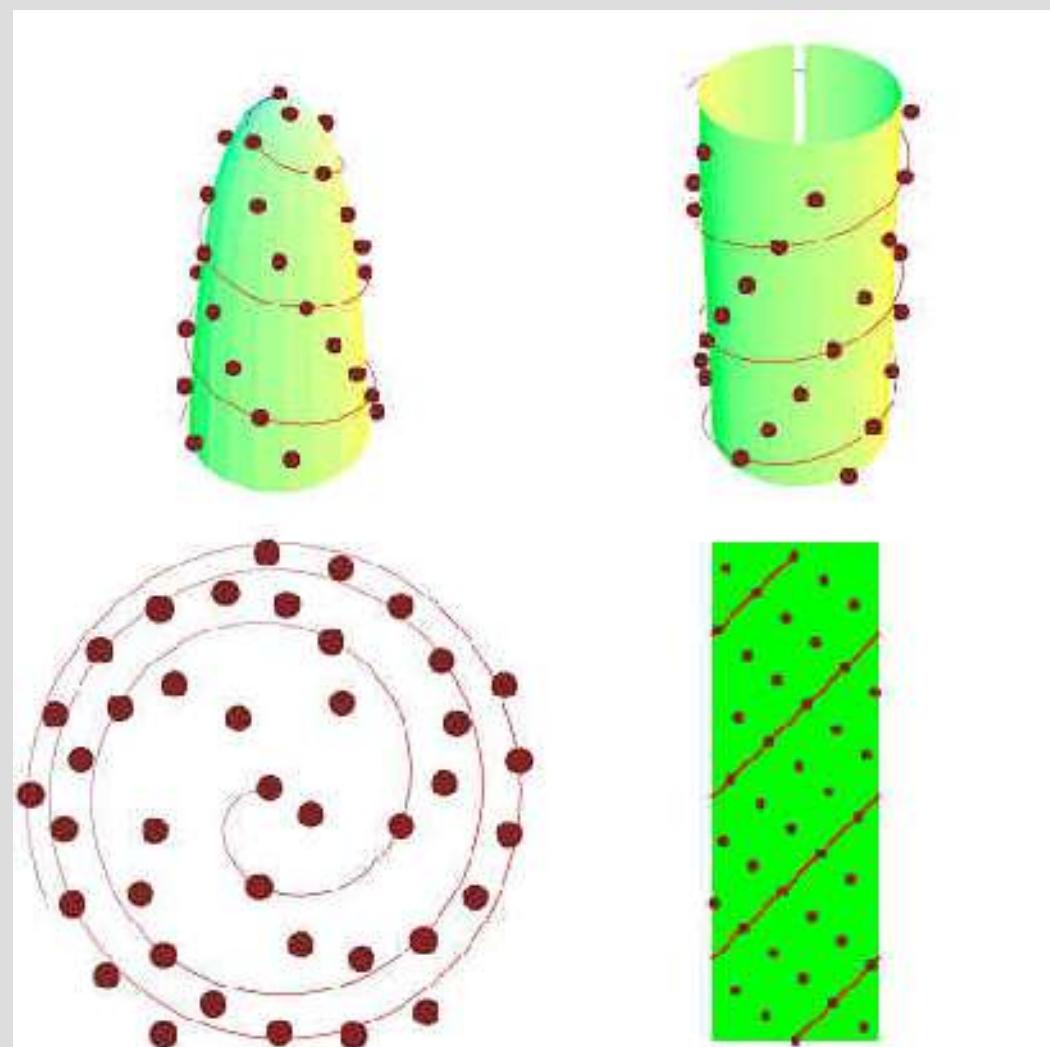
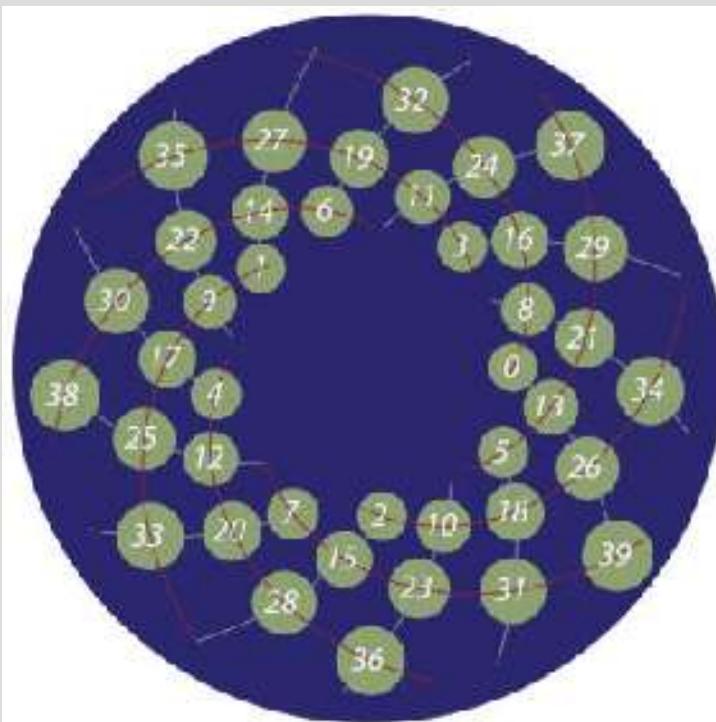


A close-up photograph of a succulent plant, likely an Aeonium, showing its characteristic growth pattern. The plant has numerous thick, fleshy leaves that are rounded and slightly pointed at the edges. They are arranged in a dense, spiraling pattern that starts from a central point and moves outwards, creating a rose-like appearance. The leaves are a vibrant green color with some darker green veins visible. The texture of the leaves appears smooth but with natural leafy patterns. The lighting is bright, highlighting the contours of the leaves and the overall organic shape of the plant.

Spiral

Two spirals...

- paristiche: spirals made up of next neighbours
- generative spiral

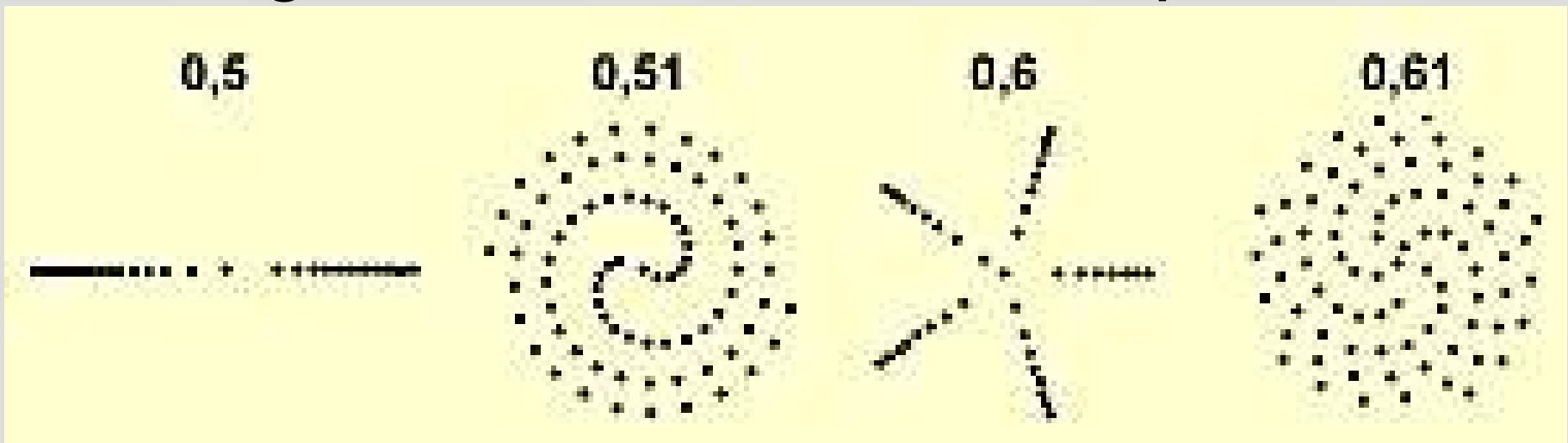


Why the golden angle?

Try different angles with the
Spiral lattices applet:

<http://www.math.smith.edu/phyllo//Applets/Spiral/Spiral.html>

Golden angle gives the densest
arrangement with least overlap



The most irrational number

There is a „most irrational“ number, and it turns out (surprise, surprise) the golden number.

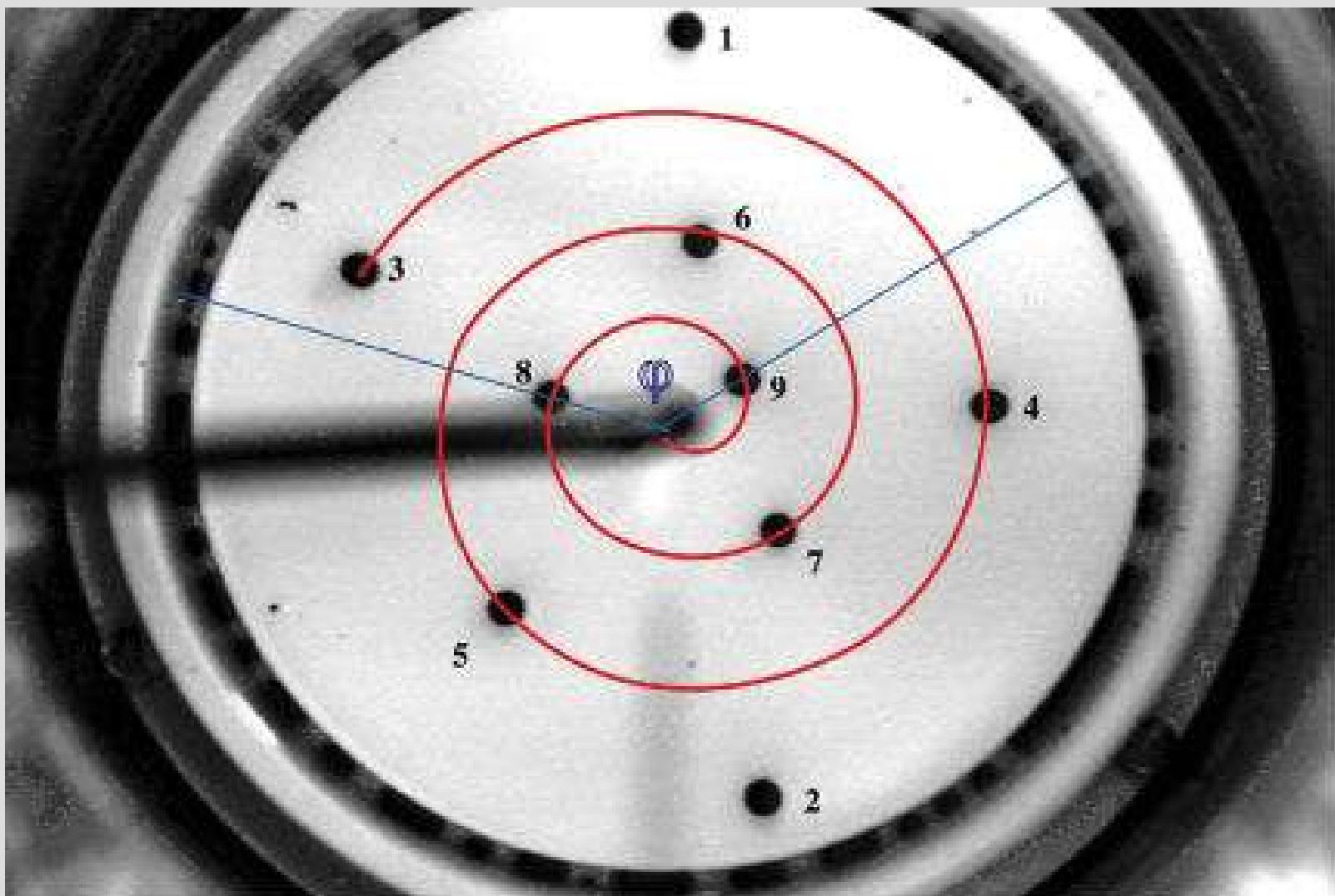
...

It is the most badly „approximable-by-rational“ number there is!

Its „badness“ is exceeded only by the awkwardness of the preceding sentence.

(Adam, Mathematics in Nature, p220)

An Experiment: Phyllotaxis as self-organization



Douady and Couder 1996

Patterns in Nature

Outline

1. Introduction
2. Waves and oscillations
3. Regularity and chaos
4. Animal cooperation
5. Spatial patterns
6. Aggregation and growth processes
7. Cellular automata
8. Fractals
9. Miscellaneous topics
10. Concluding session

