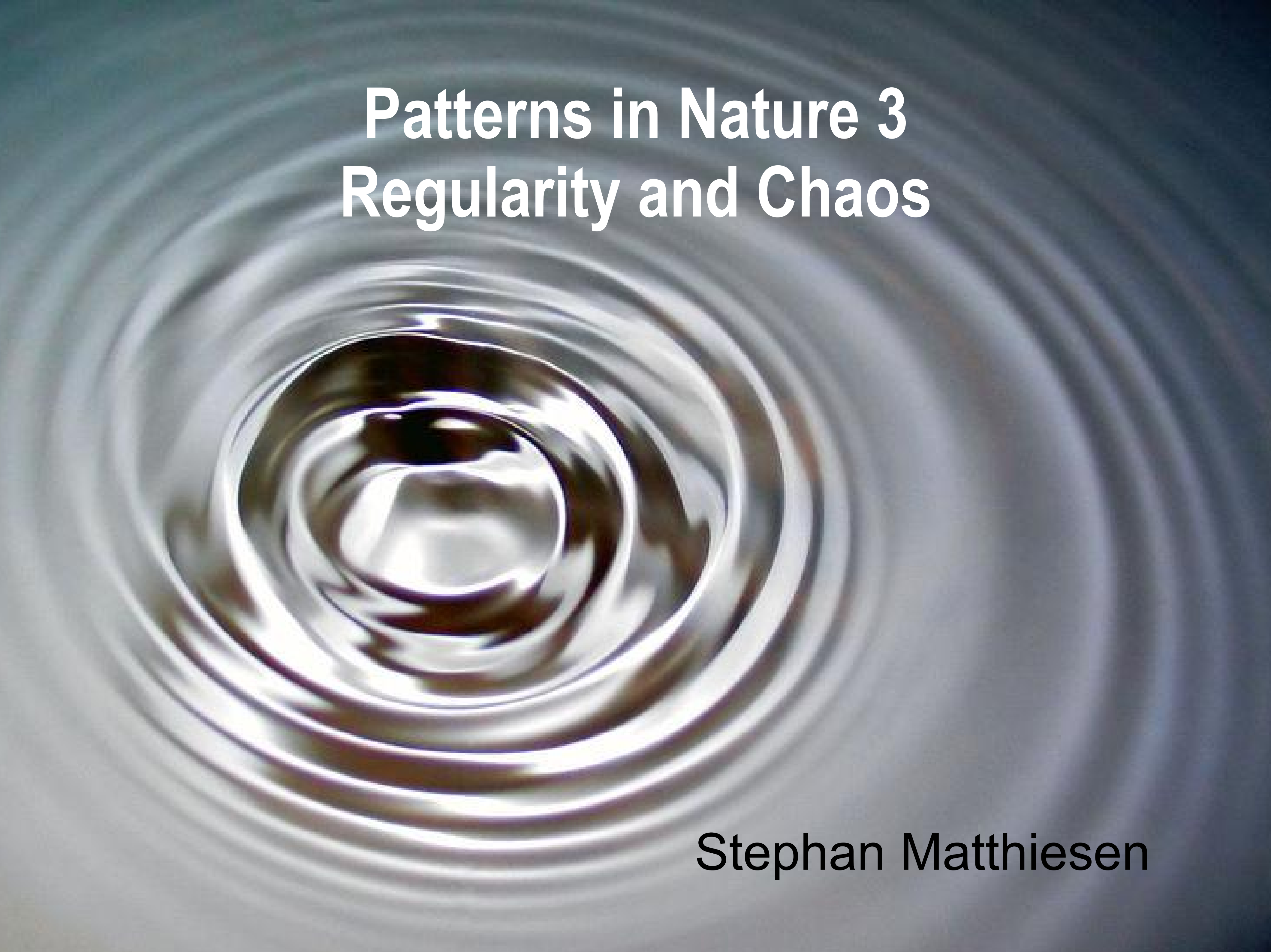


Patterns in Nature 3
Regularity and Chaos

Stephan Matthiesen

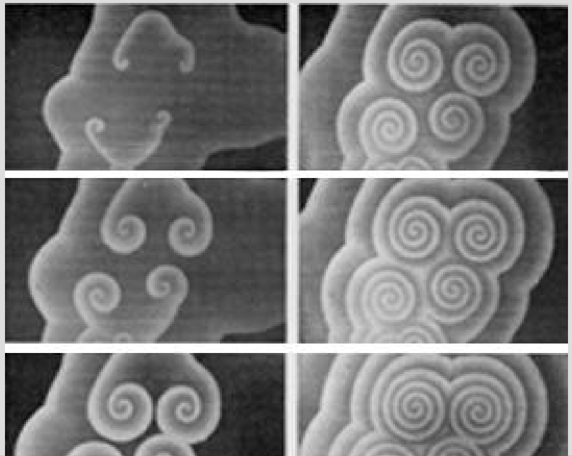
Two types of waves

„Normal“ waves

- Mechanism: restoring force
- Circular shapes
- Interaction
  - Superposition
  - Diffraction

Excitation waves

- Mechanism: excitation/latency
- Spiral shapes
- Interaction
  - Extinction

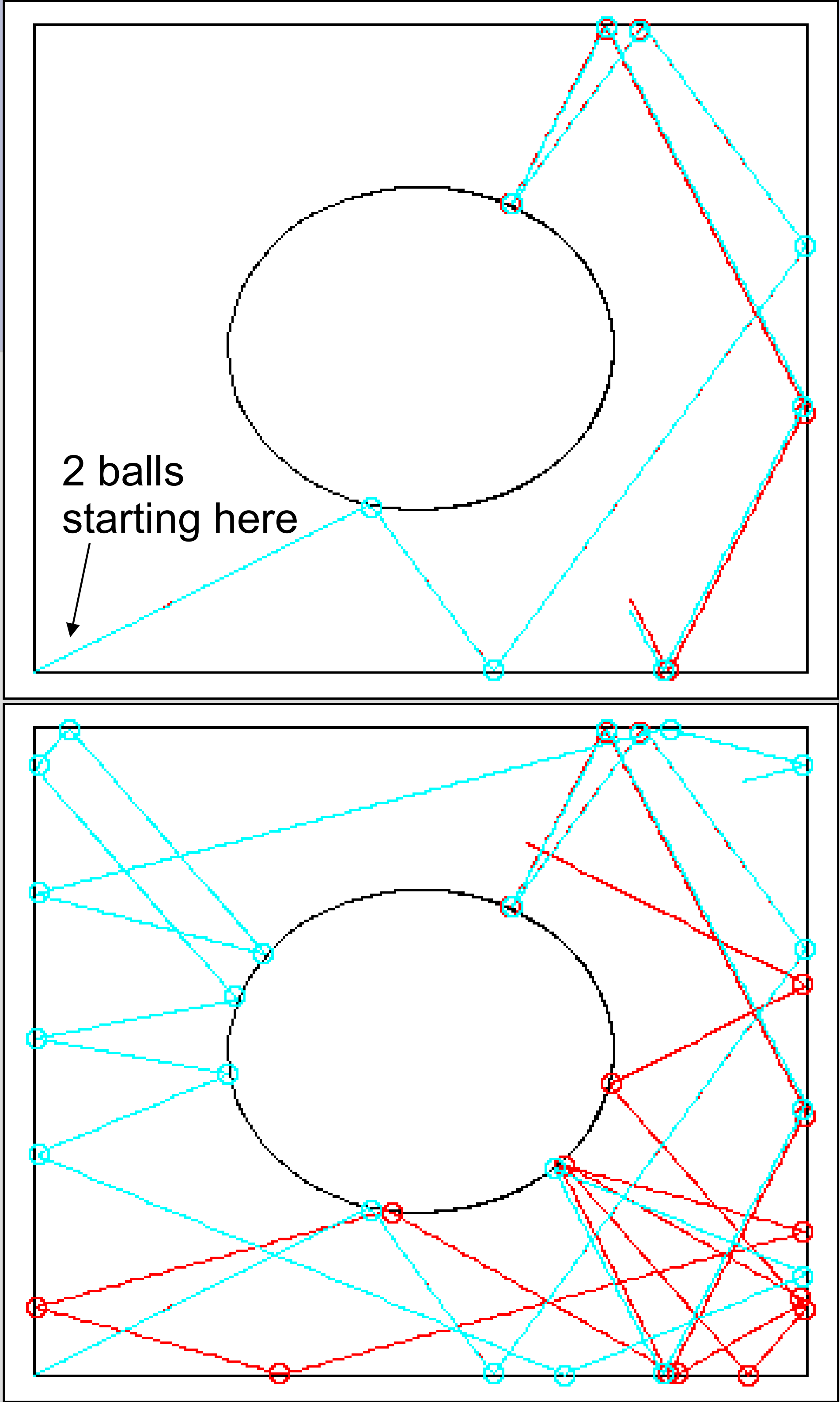


Chaos: The Sinai Billiard

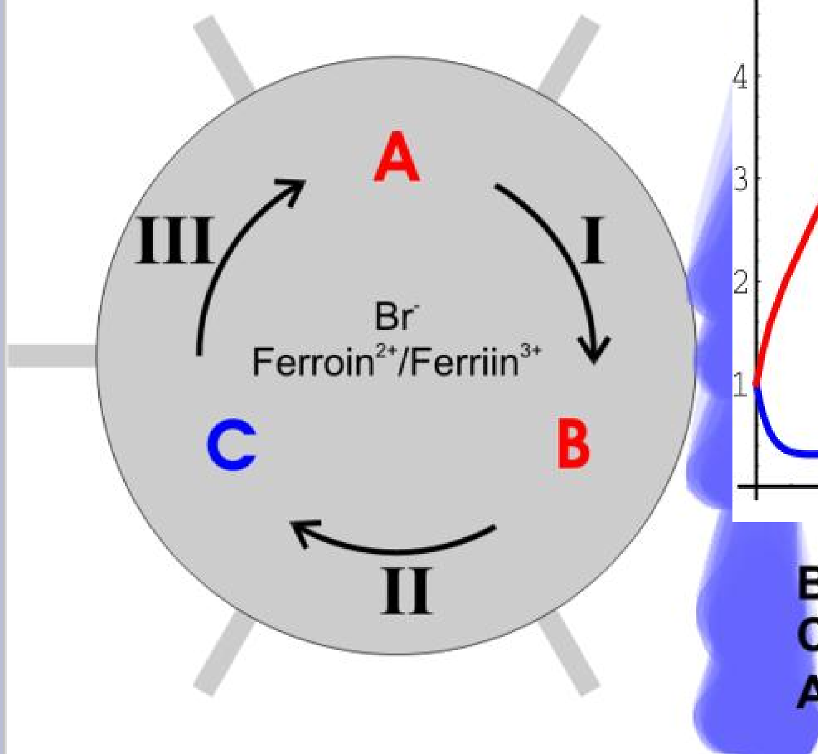
- Demonstrates unpredictability of a simple deterministic system
- Developed by Yakov G. Sinai

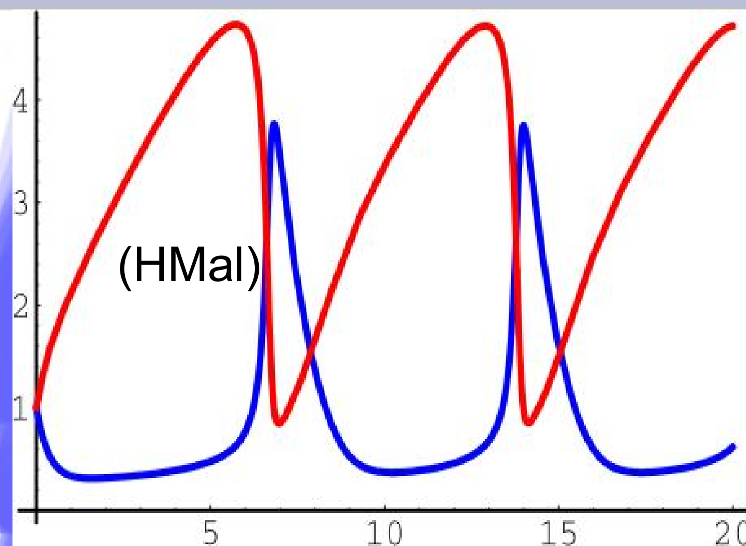
The Sinai Billiard

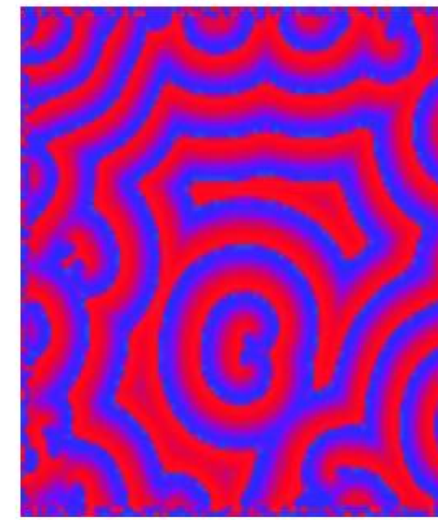
- Start two (or more) billiard balls with almost exactly the same initial conditions
- with only straight walls, their trajectories would remain close together
- the curved wall amplifies small differences (in a “nonlinear” way), the trajectories diverge fast
- **unpredictability:**
  even small (unavoidable) uncertainties lead to large differences in the final state



The Belousov-Zhabotinsky reaction







Brommalonsäure (HBrMal)
CO_2
Ameisensäure (HCOOH)

I $2\ Br^- + BrO_3^- + 3\ H^+ + 3$ "H_2Mal" $\rightarrow 3$ "HBrMal" $+ 3\ H_2O$

II $BrO_3^- + 4$ Ferroin^{2+} + "H_2Mal" $+ 5\ H^+ \rightarrow 4$ Feriin^{3+} + "HBrMal" $+ 3\ H_2O$

III 4 Feriin^{3+} + "HBrMal" $+ 2\ H_2O \rightarrow 4$ Ferroin^{2+} + HCOOH $+ 2CO_2 + 5\ H^+ + Br^-$

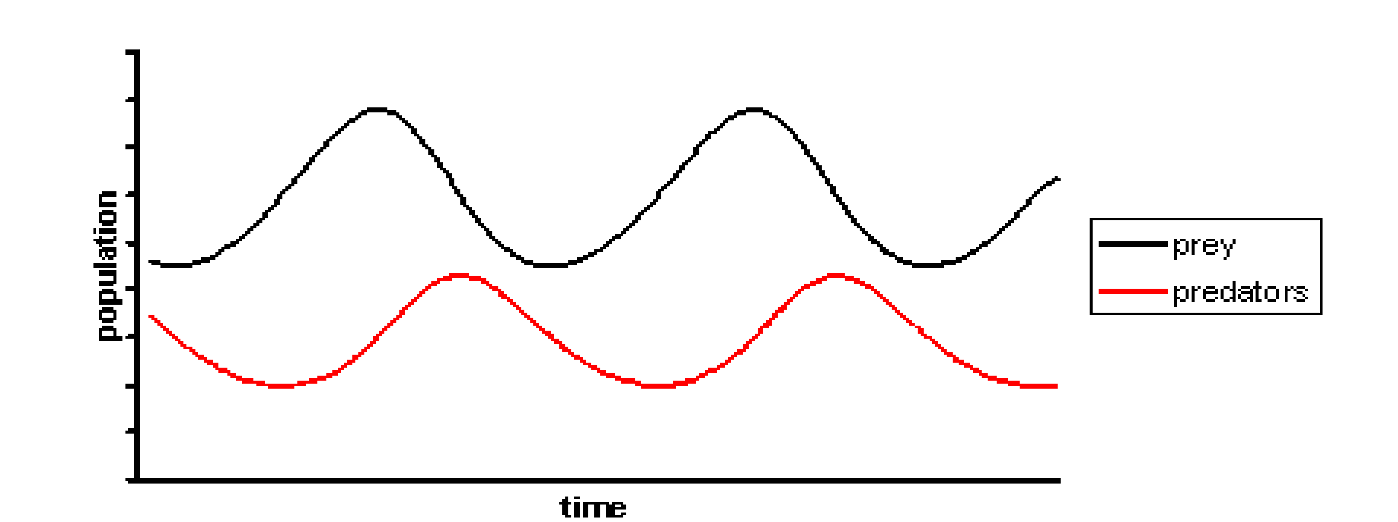
IV $3\ BrO_3^- + 5$ "H_2Mal" $+ 3\ H^+ \rightarrow 3$ "HBrMal" + 2 HCOOH $+ 4\ CO_2 + 5\ H_2O$

Reaction II is inhibited by Br-

Rabbits and foxes:
The Lotka-Volterra model

Rabbits and foxes on an isolated island:

- Rabbits and grass lead to more rabbits
- Rabbits and foxes lead to more foxes (and fewer rabbits)
- Foxes lead to some dead foxes



http://en.wikipedia.org/wiki/Lotka-Volterra_equation

The logistic map

Developed by (Lord) Robert May (1976)

A simple population model of one species:
(eg. rabbits on a small island)

- when population is low:
  population increases proportional to current population
- when population is large:
  starvation, population decreases

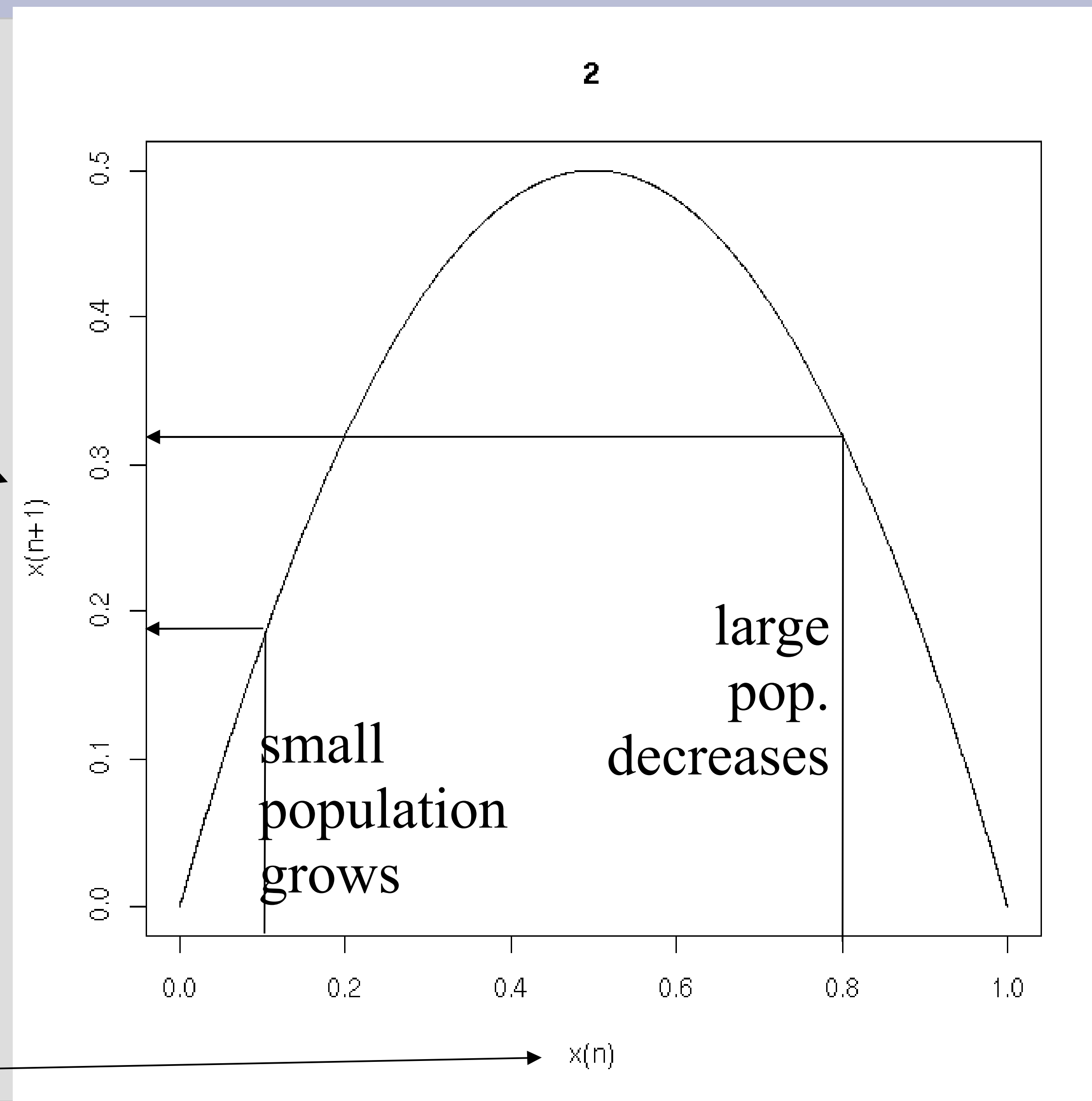
The logistic map

population in the next timestep

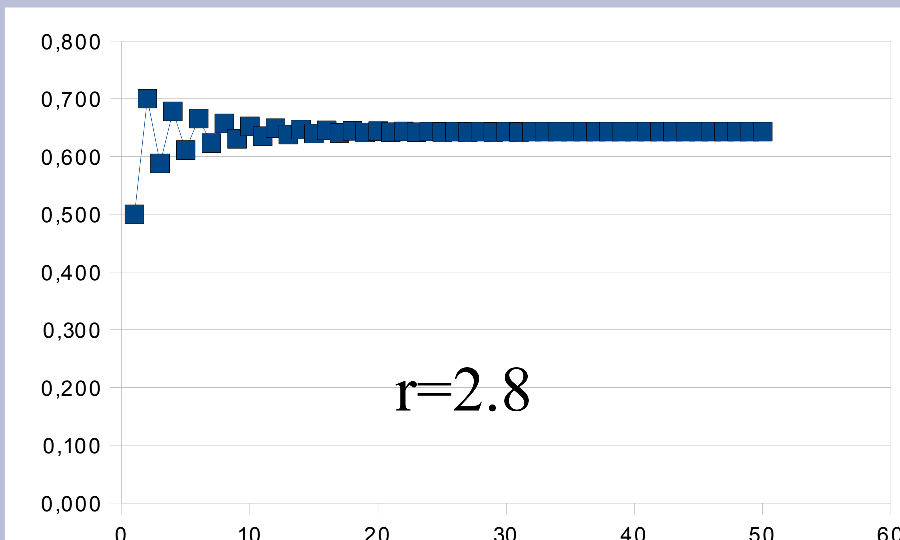
reproduction parameter

$$x_{n+1} = r\,x_n(1 - x_n)$$

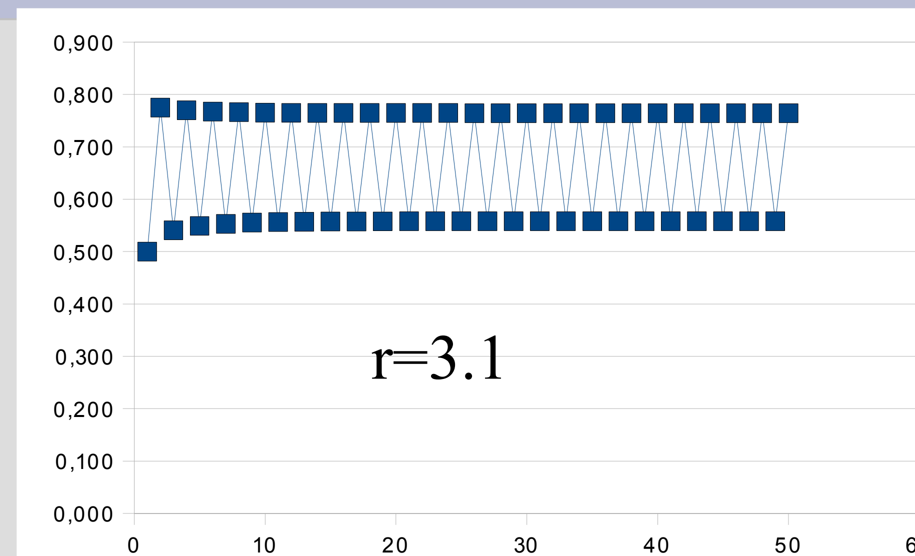
current population



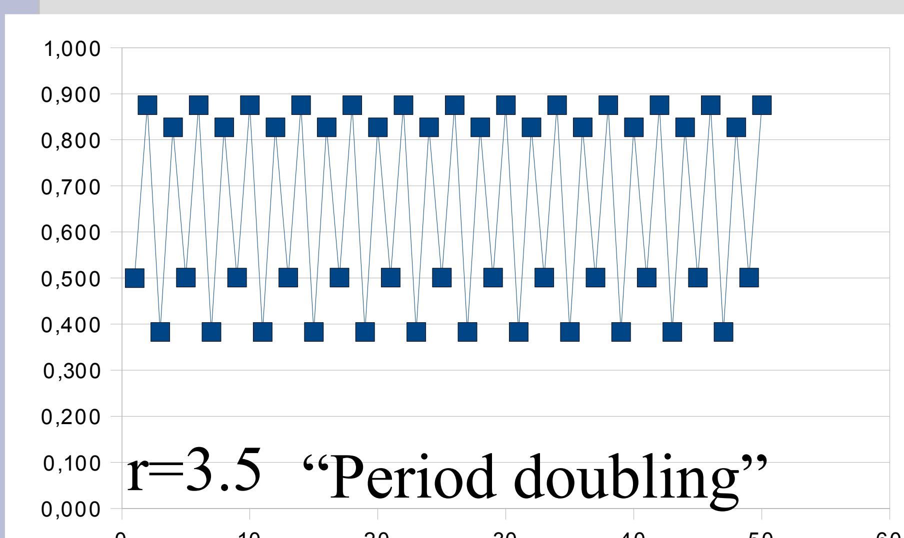
The logistic map (modelled with a spreadsheet)

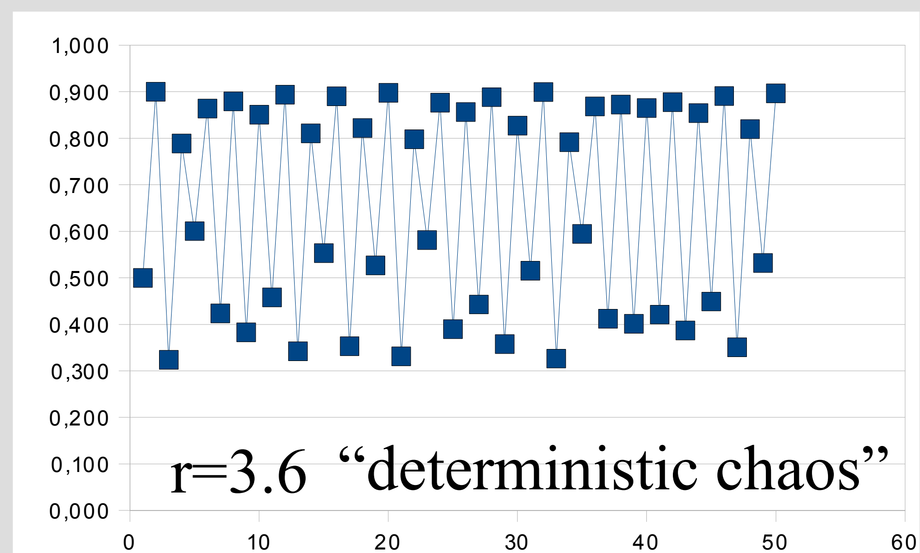


Time

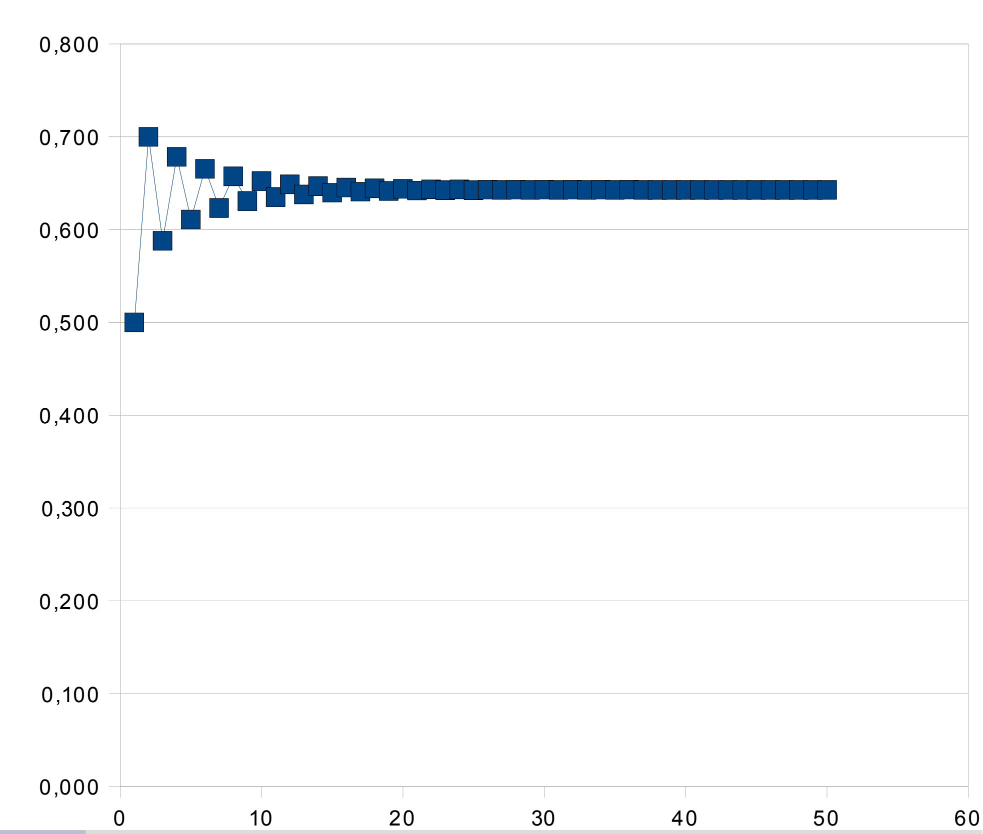


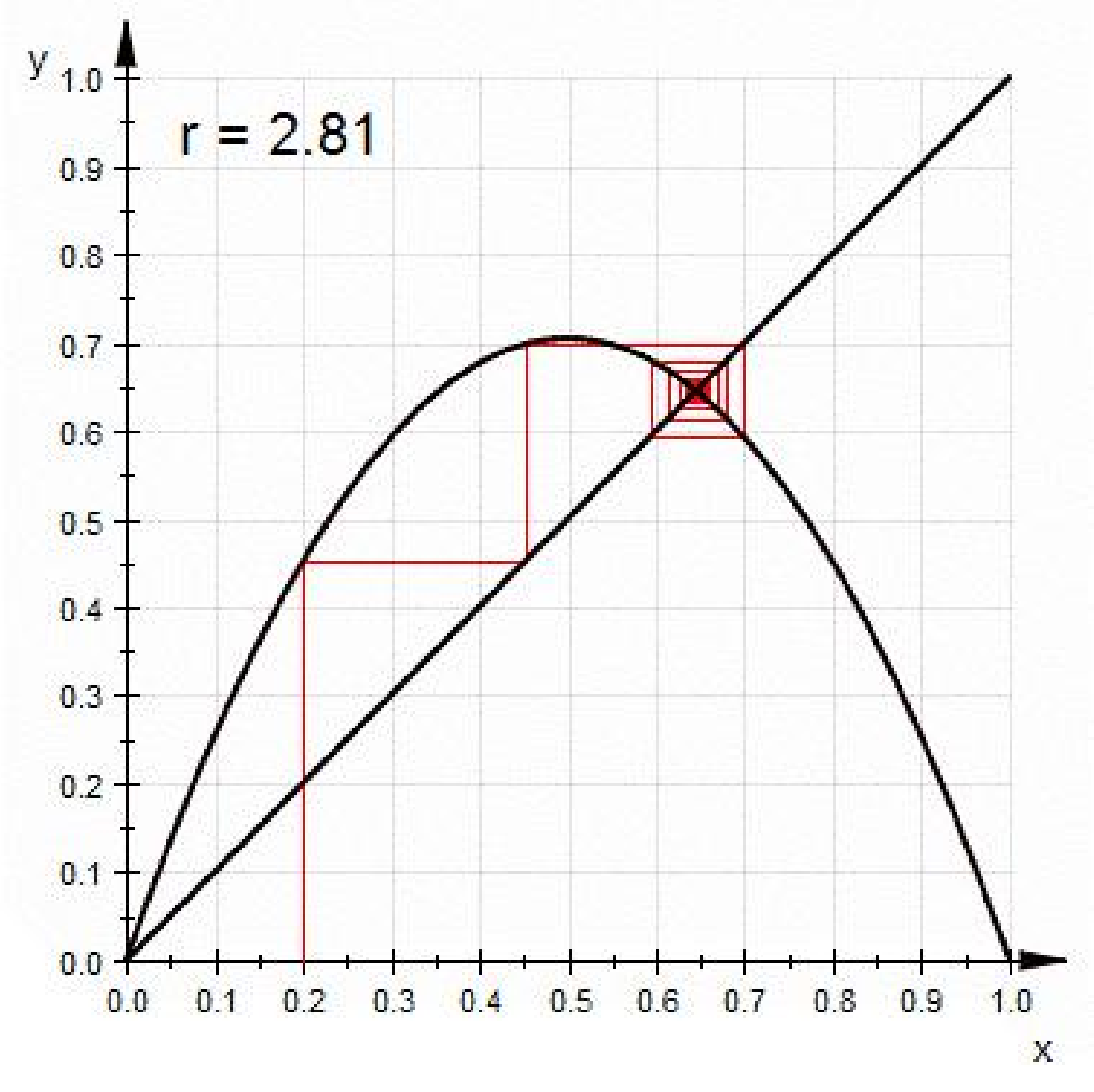
Time



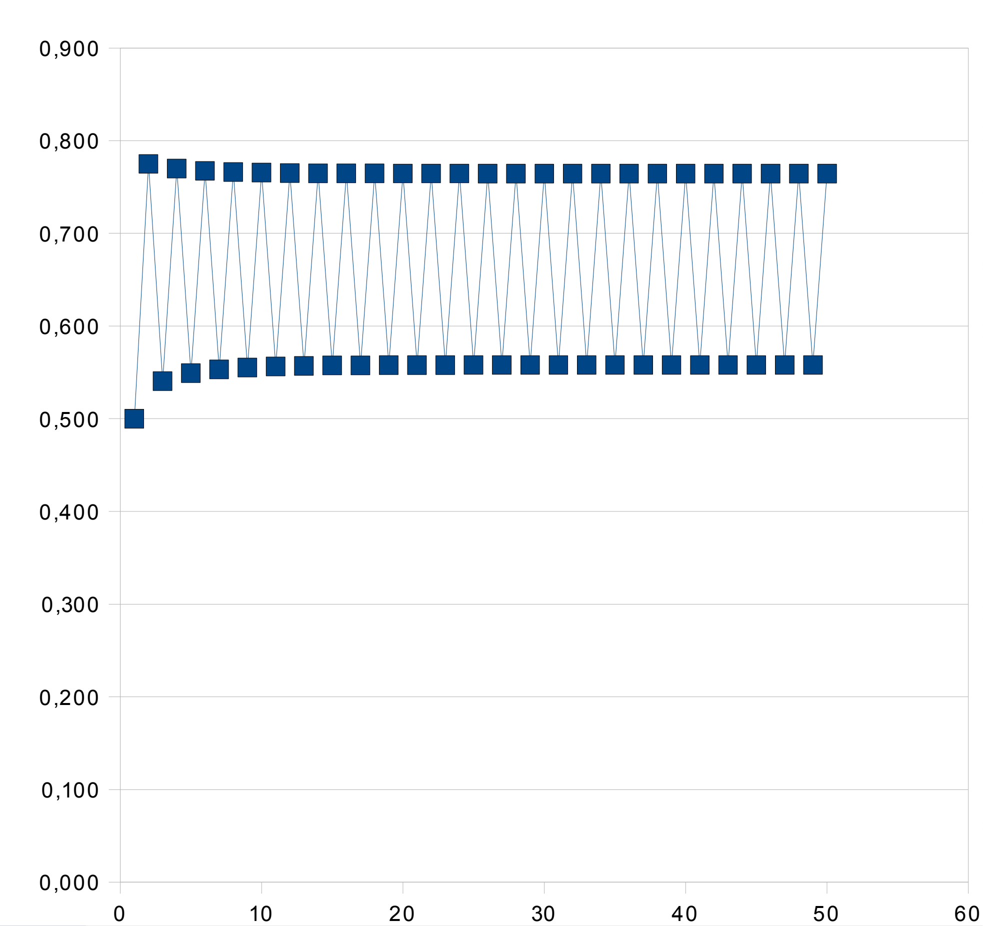


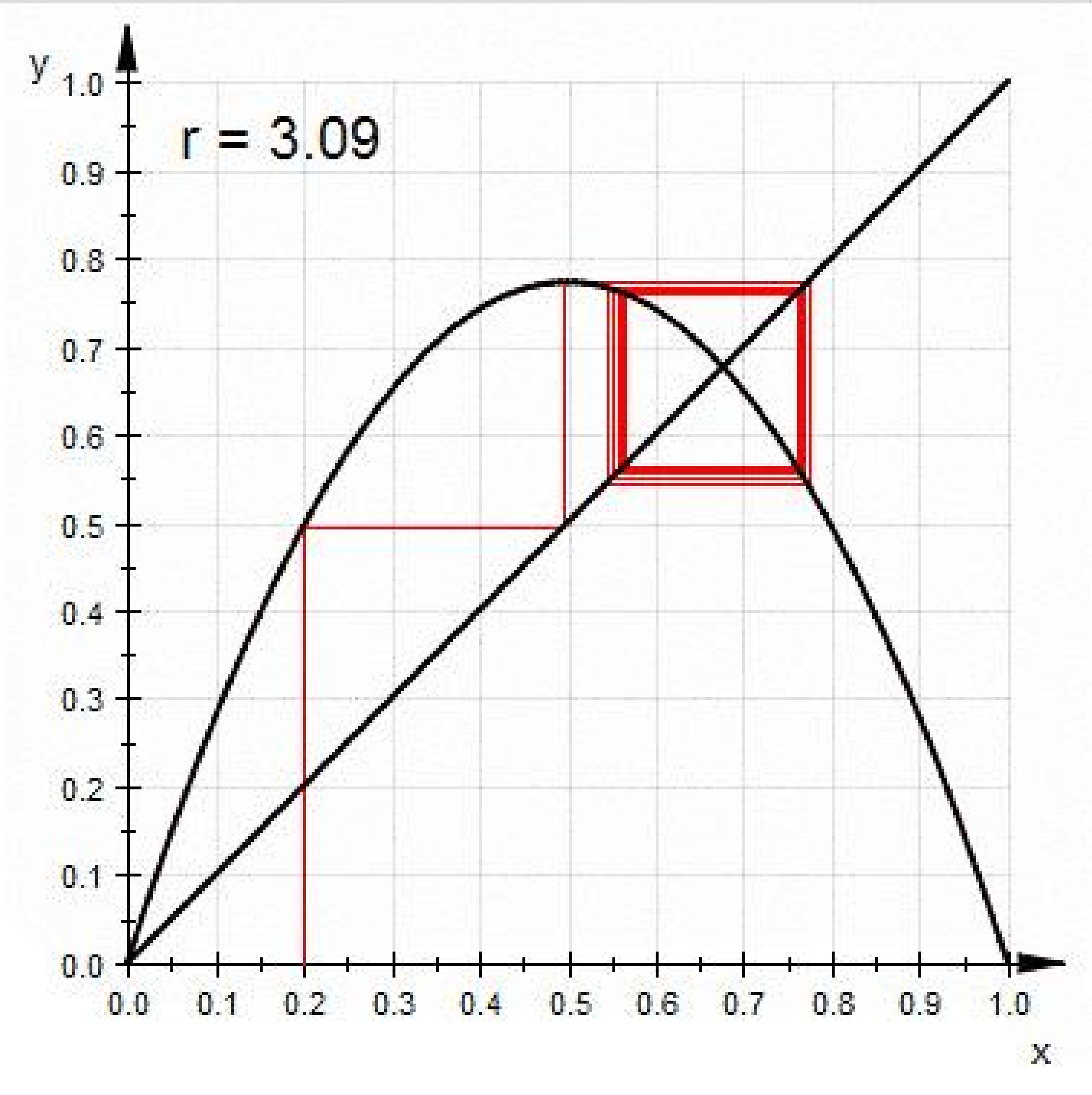
Logistic Map



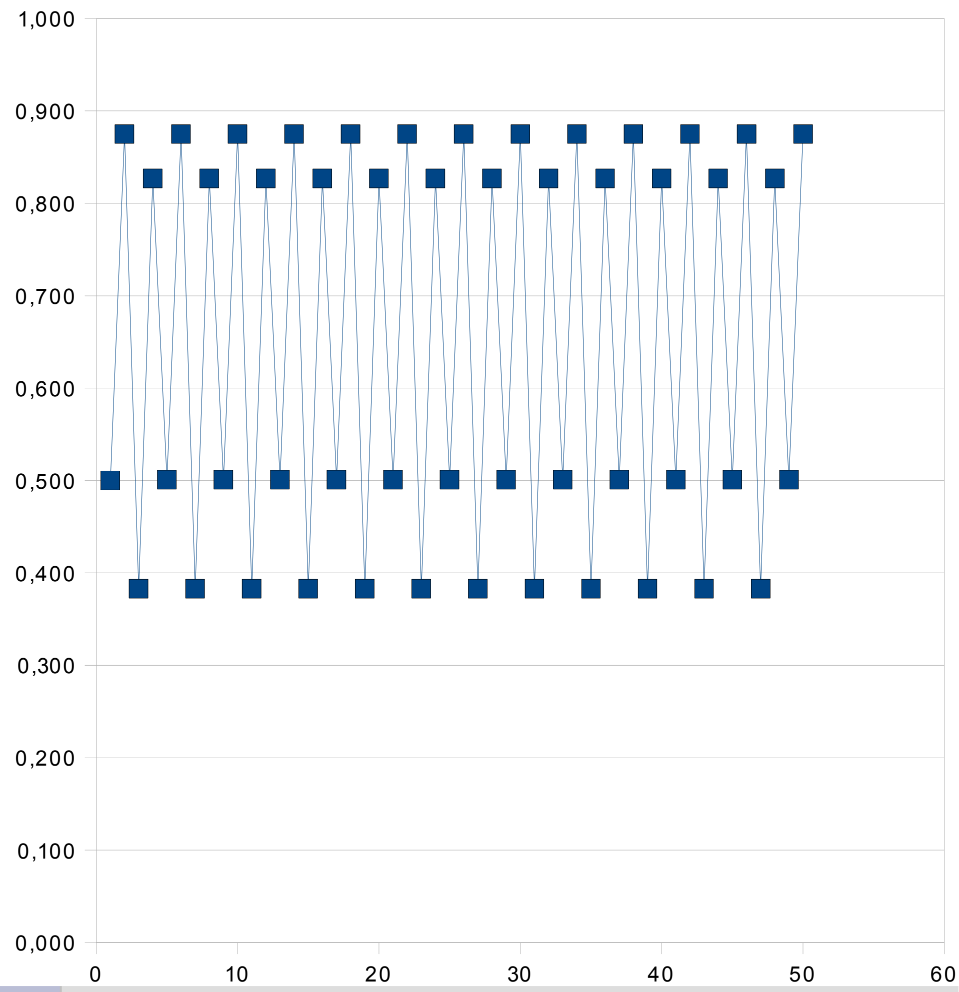


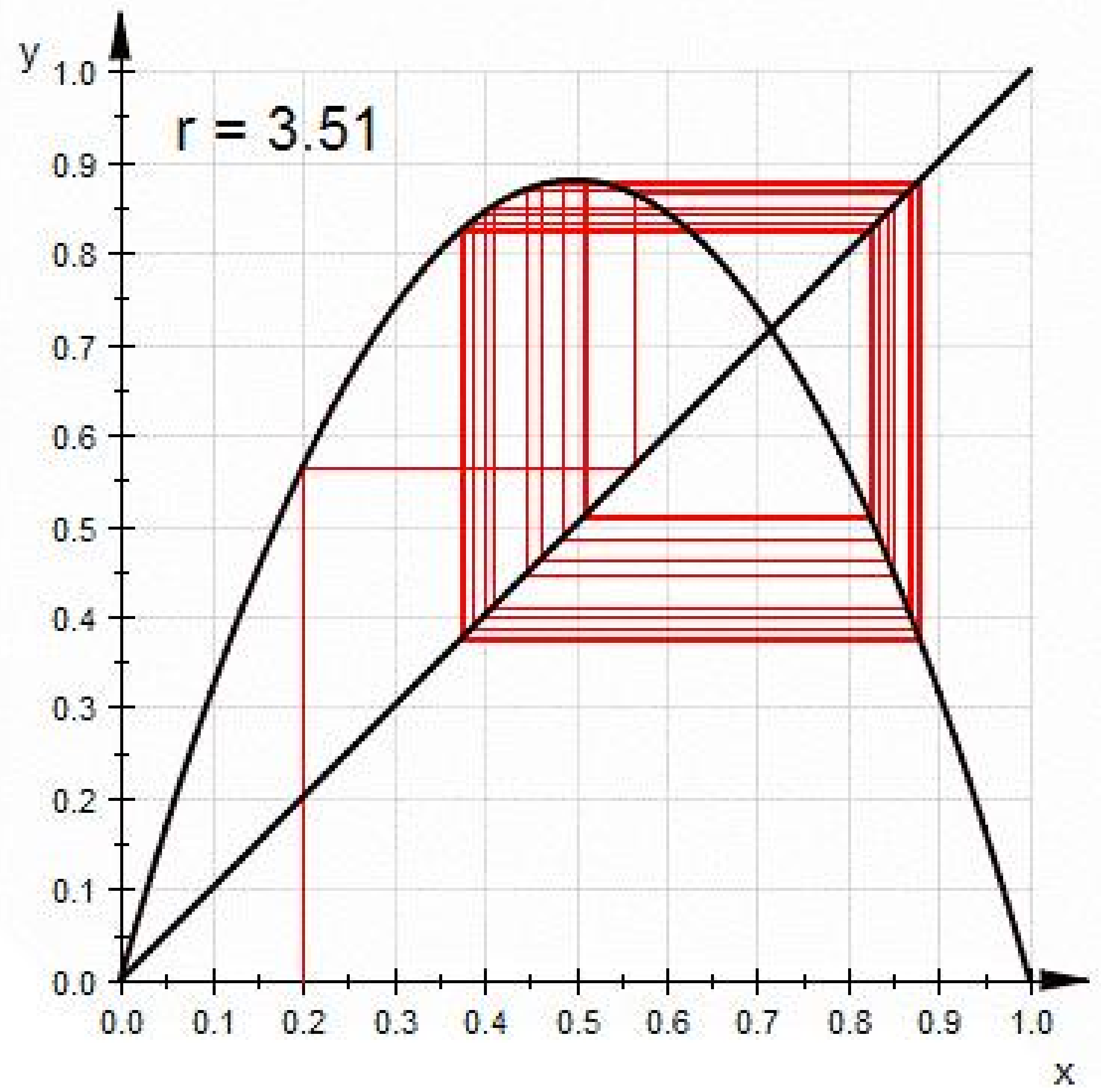
Logistic Map



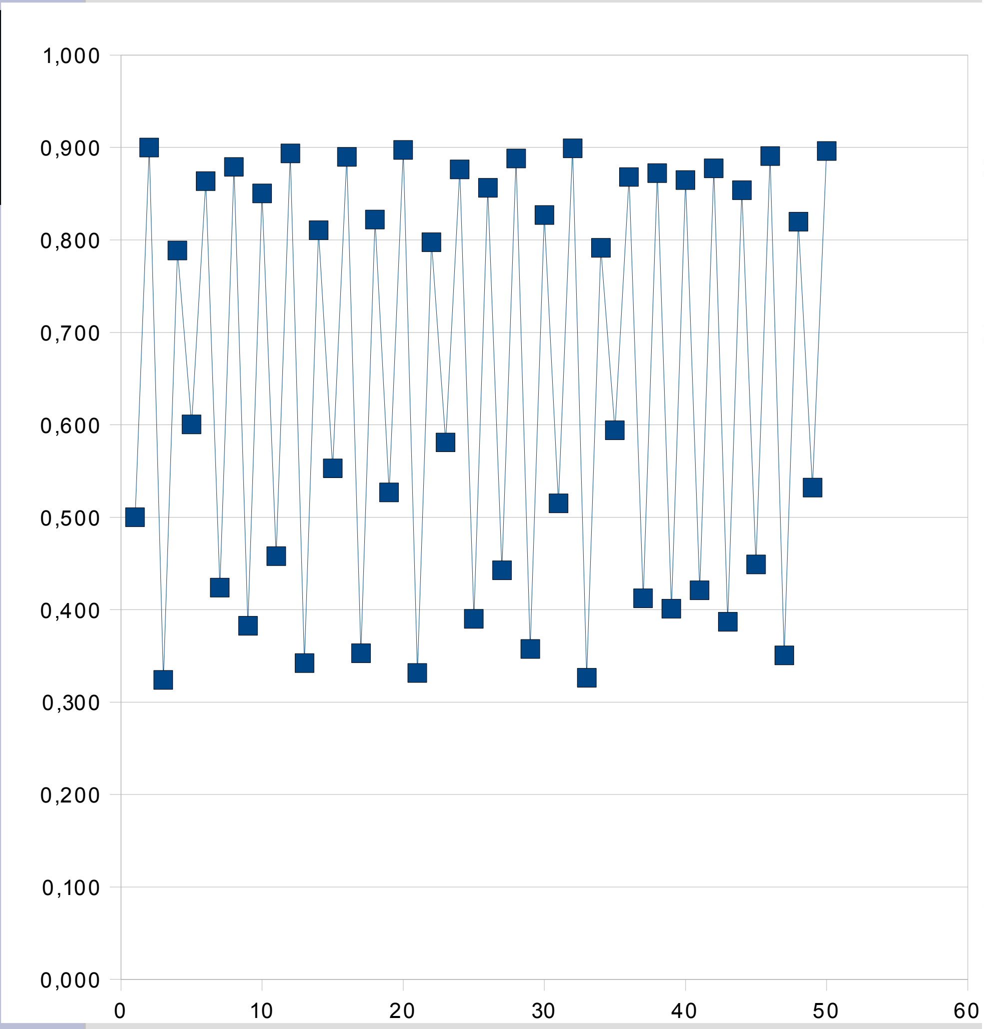


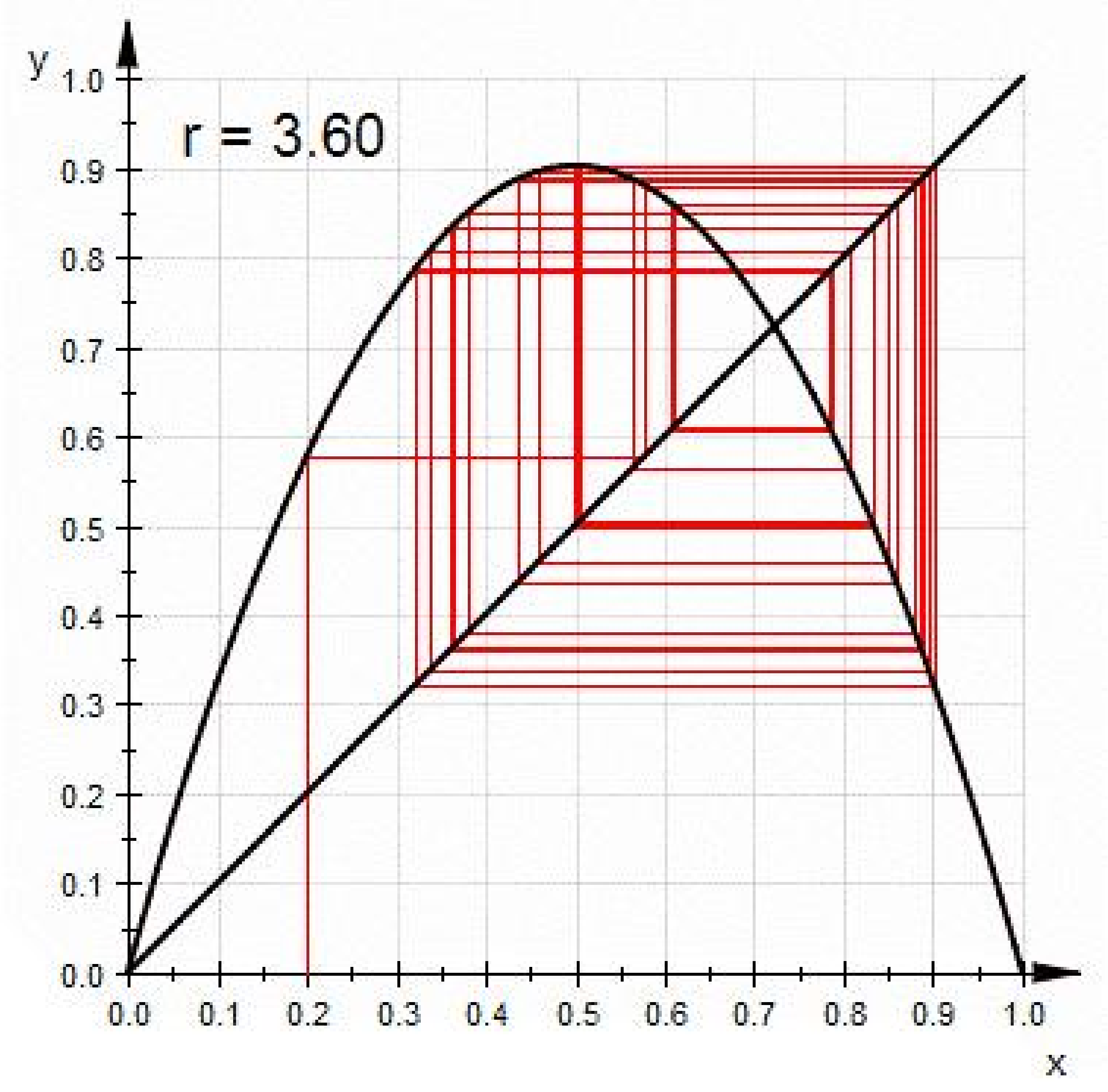
Logistic Map



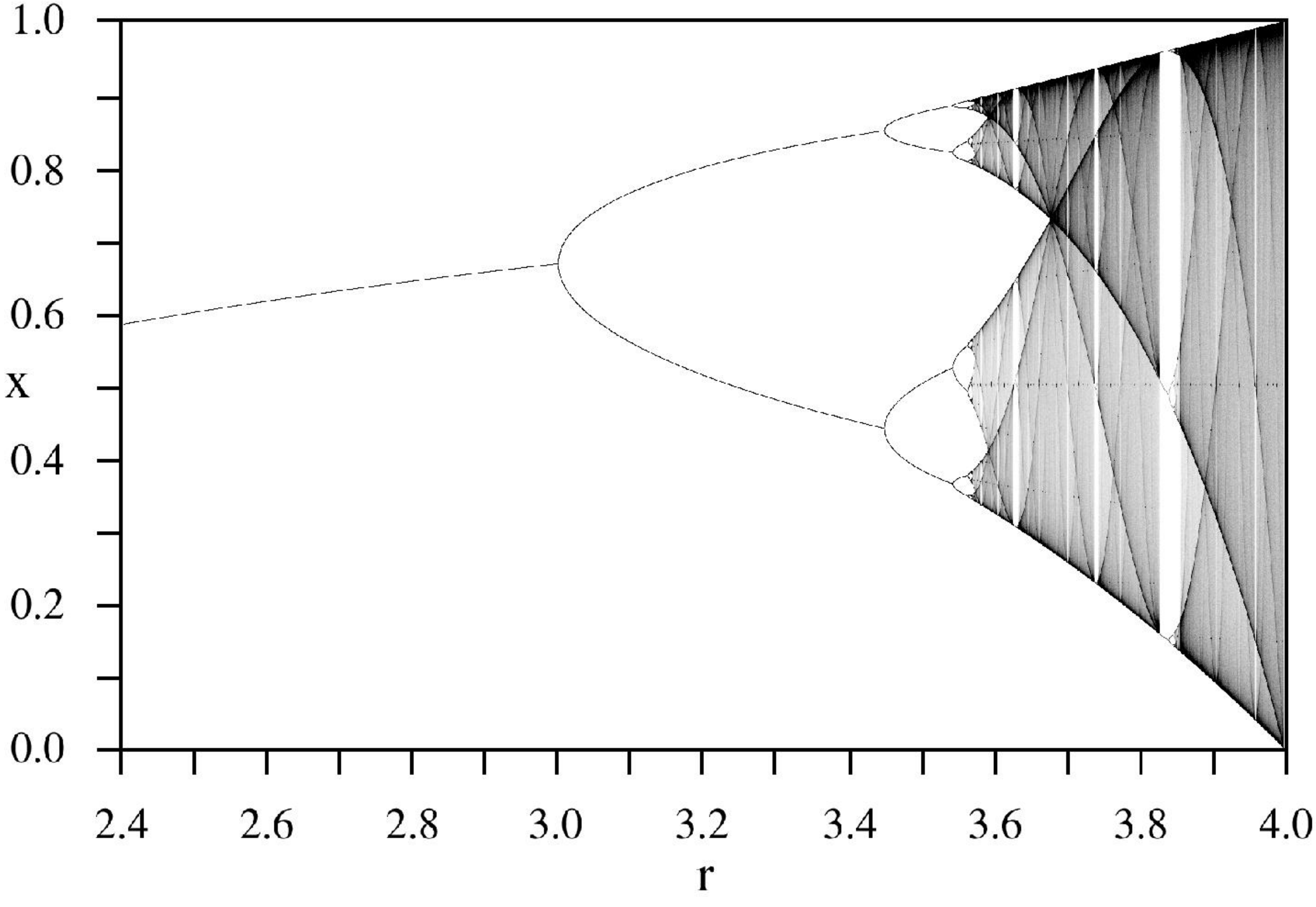


Logistic Map





Bifurcation diagram



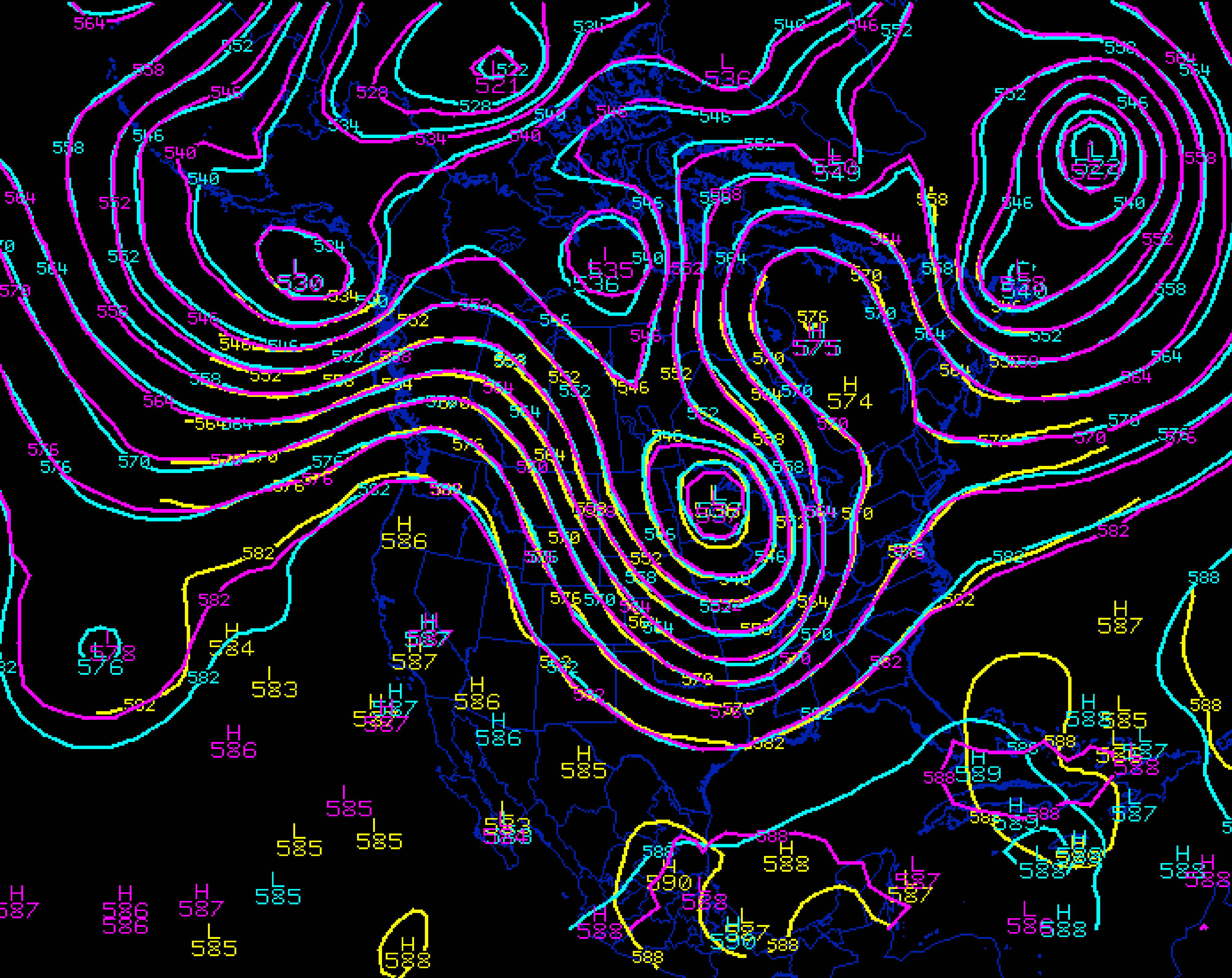
Some terms in the theory of nonlinear dynamical systems

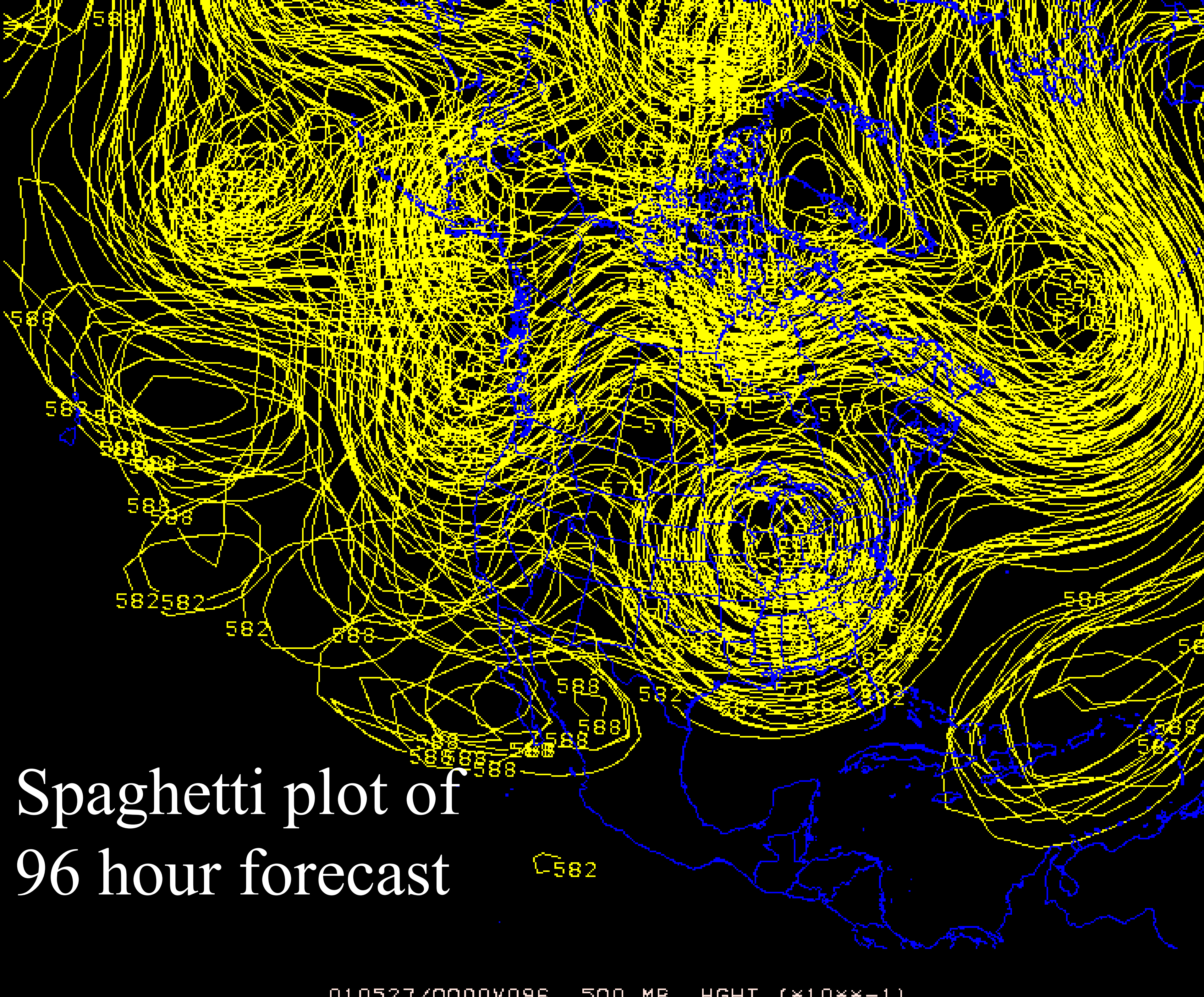
- Nonlinear dynamical system
- Attractor: The state that the system moves towards
- Strange Attractor: An attractor that is not a simple point/value
- Deterministic chaos: non-periodicity in a deterministic system (one that doesn’t include random influences)

The weather

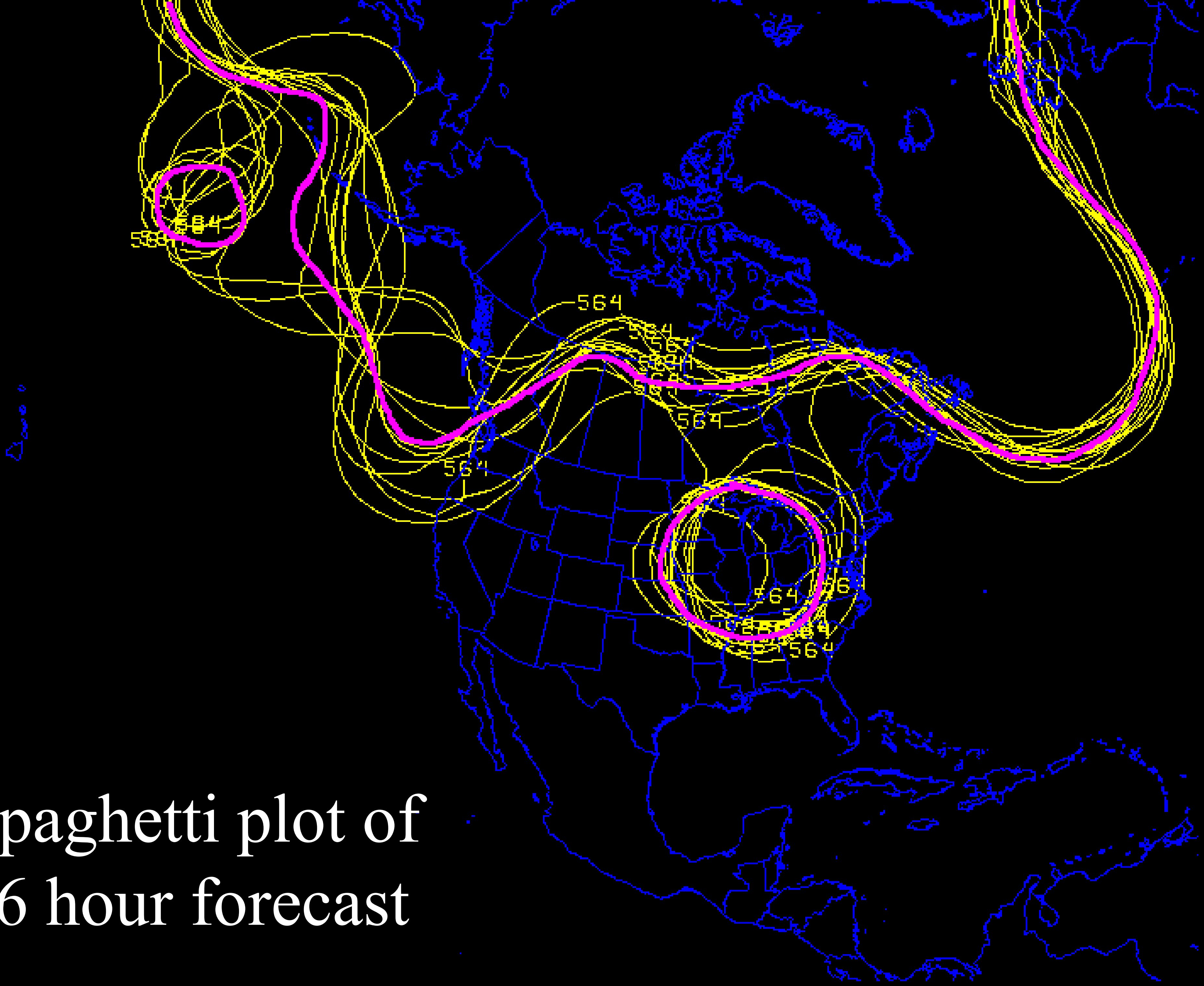
- Weather is described with complicated equations (much more complicated than the logistic map)
- As these equations are “nonlinear”, we expect unpredicability (in the sense explained above)

=> Use **ensemble predictions**
(i.e. run the forecast 50 times with small disturbances and see if the results differ)
(following examples from http://www.hpc.ncep.noaa.gov/ensembletraining/)





Spaghetti plot of
96 hour forecast



Spaghetti plot of 96 hour forecast

Weather

- Some situations are more „chaotic“ than others
- Projections of climate are easier, because climate variables are averages
  Analogy: when throwing dice, you can't predict the next number, but you can predict that among the next 600 numbers there will be approximately 100 number 6

Patterns in Nature
Outline

1. Introduction
2. Waves and oscillations
3. Regularity and chaos
4. Animal cooperation
5. Spatial patterns
6. Aggregation and growth processes
7. Cellular automata
8. Fractals
9. Miscellaneous topics
10. Concluding session