Atmospheric Dynamics Lect.15: Baroclinic Rossby waves

Stephan Matthiesen

Institute for Atmospheric and Environmental Sciences

School of Geosciences, Edinburgh University

Crew Building room 218

Scope of Lecture

- ln(pressure) as vertical co-ordinate
- Linearised equation
- Preliminary features of the solution

ln(p) co-ordinates

- p has some disadvantages as co-ordinate
- -ln(p) is more natural, but a useful scaling is given by the height-like variable

$$z^* \equiv H_{00} \ln\left(\frac{p_{00}}{p}\right) \qquad \textbf{with} \qquad H_{00} = \frac{RT_{00}}{g}$$

$$w^* \equiv \frac{Dz^*}{Dt} = -H_{00}\frac{1}{p}\frac{Dp}{Dt} = -\frac{H_{00}\,\omega}{p}$$

Conservation of q-g potential vorticity in ln(p) co-ordinates

- p co-ordinate version

$$\left(\frac{\partial}{\partial t}+u_g\frac{\partial}{\partial x}+v_g\frac{\partial}{\partial y}\right)_p q_g=0 \qquad q_g\equiv\zeta_{rel}+f+\frac{f_0^2}{R}\frac{\partial}{\partial p}\left(\frac{p}{\Gamma_0}\frac{\partial\Psi}{\partial p}\right)$$

- ln(p) co-ordinate version

$$q_g=\nabla^2\Psi+f+\left(\exp\frac{z^*}{H_{00}}\right)\frac{\partial}{\partial z^*}\left\{\frac{f_0^2}{N^{*2}}\left(\exp\frac{-z^*}{H_{00}}\right)\frac{\partial\Psi}{\partial z^*}\right\}$$

where $N^*\equiv\sqrt{\frac{H}{H_{00}}\frac{g}{\theta}\frac{\partial\theta}{\partial z^*}}$ is a modified Brunt-Vaisaila frequency

Application to simple case

Treat case with constant u_0 and N^*

Conservation of q-g potential vorticity linearises to

$$\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial x}\right)\left[\nabla^2\Psi'+\frac{f_0^2}{N^{*2}}\frac{\partial^2\Psi'}{\partial z^{*2}}-\frac{f_0^2}{H_{00}N^{*2}}\frac{\partial\Psi'}{\partial z^*}\right]+\beta\frac{\partial\Psi'}{\partial x}=0$$

Seek wavelike solutions

$$\Psi'=A\exp i\left(\sigma t+\lambda x+\mu y+nz\right)$$

On substituting we find we need

$$i\left(\sigma+u_0\lambda\right)\left[-\left(\lambda^2+\mu^2\right)-\frac{f_0^2}{N^{*2}}n^2-i\frac{f_0^2}{H_{00}N^{*2}}n\right]+i\beta\lambda=0$$

Implications for vertical variation

Note that n cannot be wholly real

We need $n^2 + i\frac{n}{H_{00}}$ to be real

Set $n = \nu + i\,n_i$ with n_i and ν real

Gives requirement that $i\left(2\nu n_i + \frac{\nu}{H_{00}}\right)$ is real, i.e.

$$n_i = -\frac{1}{2\mathrm{H}_{00}}$$

Implications for vertical variation (cont)

In effect

$$\psi' = \left[A \exp \frac{z^*}{2H_{00}} \right] \exp i \left(\sigma t + \lambda x + \mu y + \nu z^* \right)$$

with

$$\nu^2 = \frac{N^{*2}}{f_0^2} \left[\frac{\beta}{u_0 + c} - \left(\lambda^2 + \mu^2 \right) \right] - \frac{1}{4H_{00}{}^2}$$

where $c = -\sigma / \lambda$

When solutions are wavelike (i.e. real λ,μ,ν)

Magnitude of ψ' is proportional to

$$\exp \frac{z^*}{2H_{00}} = \left(\frac{p_0}{p} \right)^{\frac{1}{2}}$$

Critical wind for stationary waves

For stationary waves c=0

$$\nu^2 = \frac{N^2}{f_0^2}\left[\frac{\beta}{u_0} - \left(\lambda^2 + \mu^2\right)\right] - \frac{1}{4H_{00}^2}$$

Choose waves of particular horizontal wavelengths (i.e. fix λ, μ) and introduce u_c (a certain critical windspeed) :-

$$u_c = \beta\left[\lambda^2 + \mu^2 + \left(\frac{f_0}{2NH}\right)^2\right]^{-1}$$

giving

$$\nu^2 = \frac{N^2}{f_0^2}\left[\frac{\beta}{u_0} - \frac{\beta}{u_c}\right]$$

Propagating case

For real ν we must have $0 < u_0 < u_c$

Phase is constant on surface $\left(\lambda x + \mu y + \nu z^*\right) = const$.

For given latitude phase lines are $\left(\lambda x + \nu z^*\right) = const$.

It can be shown that

- energy propagates upwards when phase lines tilt westwards with height
- therefore eastward slope not expected
- westward sloping waves transport heat polewards

Trapped case

If $u_0<0$ or $u>u_c$, then ν is wholly imaginary

$$\psi' = \left[A \exp\left(\frac{1}{2\mathrm{H}} \mp \nu_i \right) z^* \right] \exp i\left(\sigma t + \lambda x + \mu y \right)$$

Phaselines are vertical

For k.e. decreasing upwards take –ve sign

$$\psi' = \left[A \exp\left(\frac{1}{2\mathrm{H}_{00}} - \nu_i \right) z^* \right] \exp i\left(\sigma t + \lambda x + \mu y \right)$$

normally $\nu_i > \dfrac{1}{2\mathrm{H}_{00}}$ so that amplitude decreases upwards