Atmospheric Dynamics Lect.14: Barotropic Rossby waves

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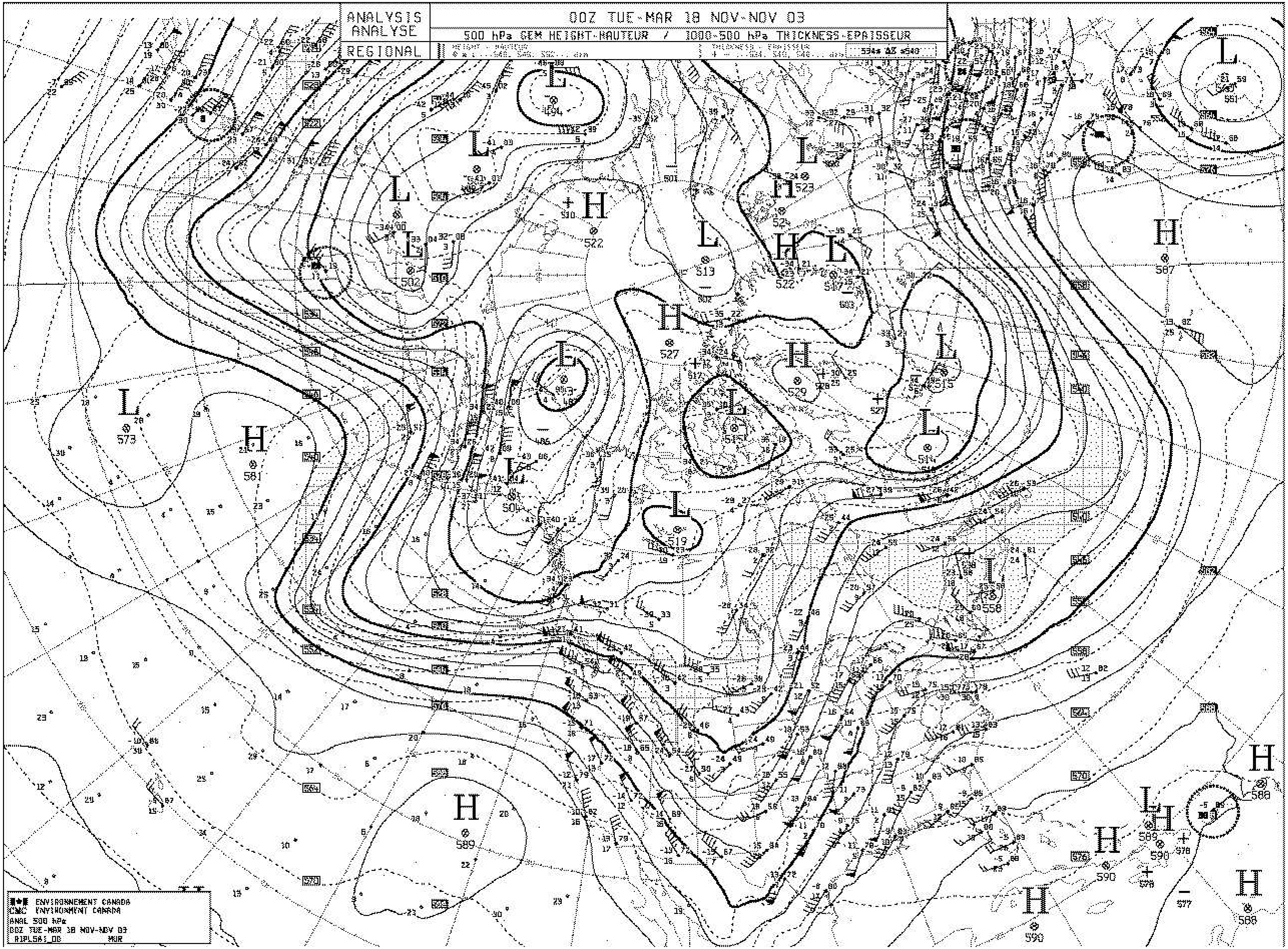
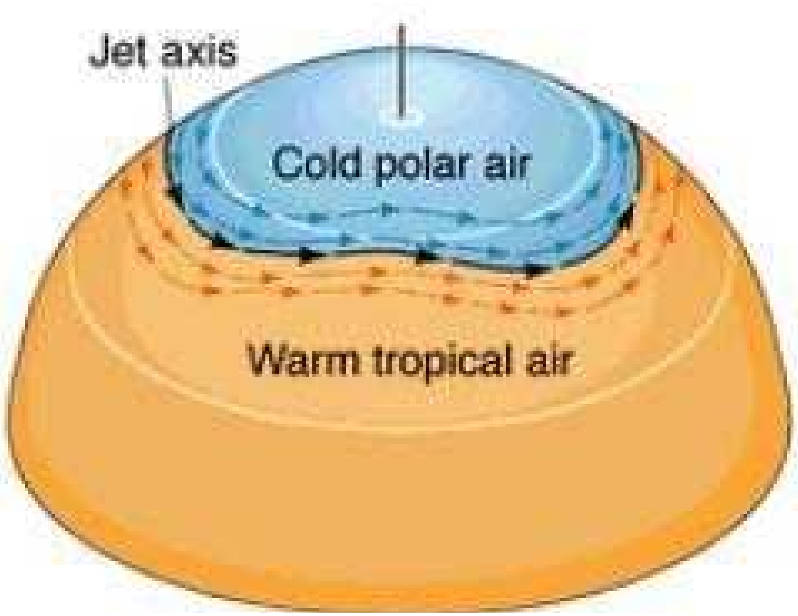
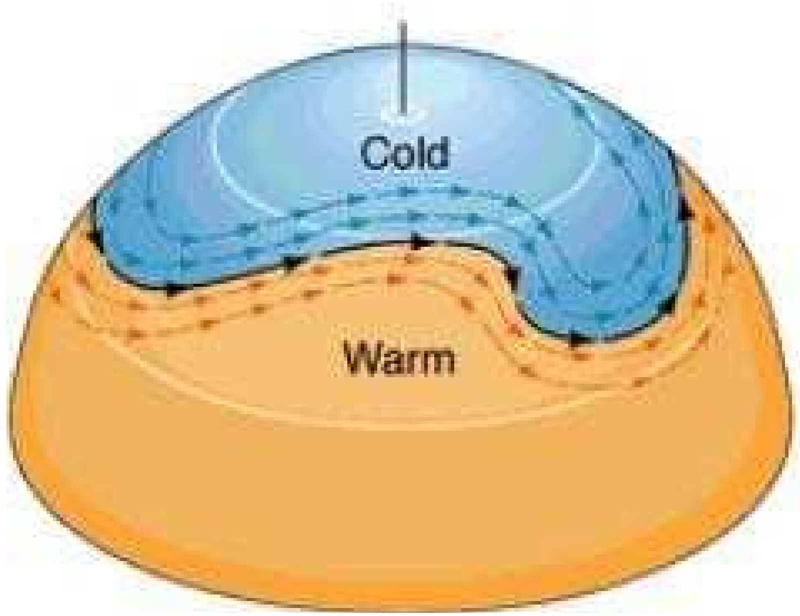
Scope of Lecture

- Some features of observed wave structure
- Theory based on parcel approach
- Meaning of barotropy
- Use of linearised wave theory to obtain phase speeds or wavelengths

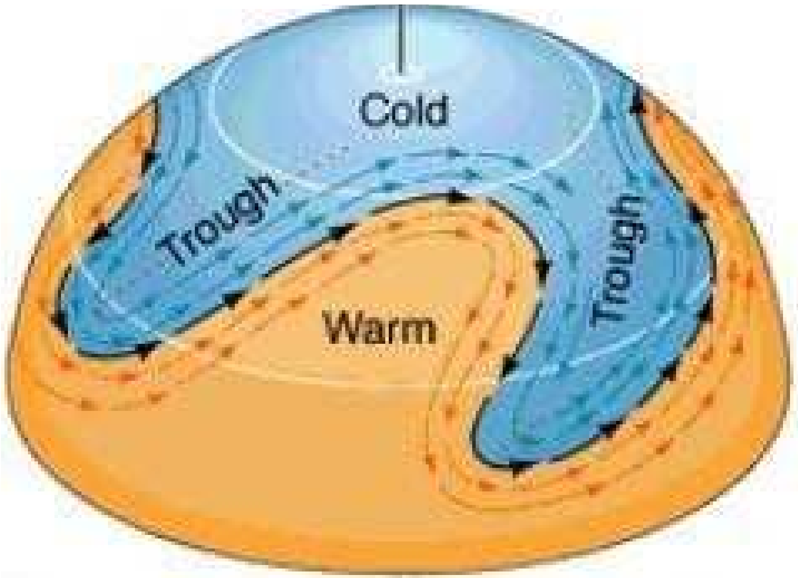

ANALYSIS
ANALYSE
REGIONAL
00Z TUE-MAR 18 NOV-NOV 03
500 hPa GEM HEIGHT-HAUTEUR / 1000-500 hPa THICKNESS-EPAISSEUR
ENVIRONNEMENT CANADA
ENVIRONMENT CANADA
ANAL 500 hPa
00Z TUE-MAR 18 NOV-NOV 03



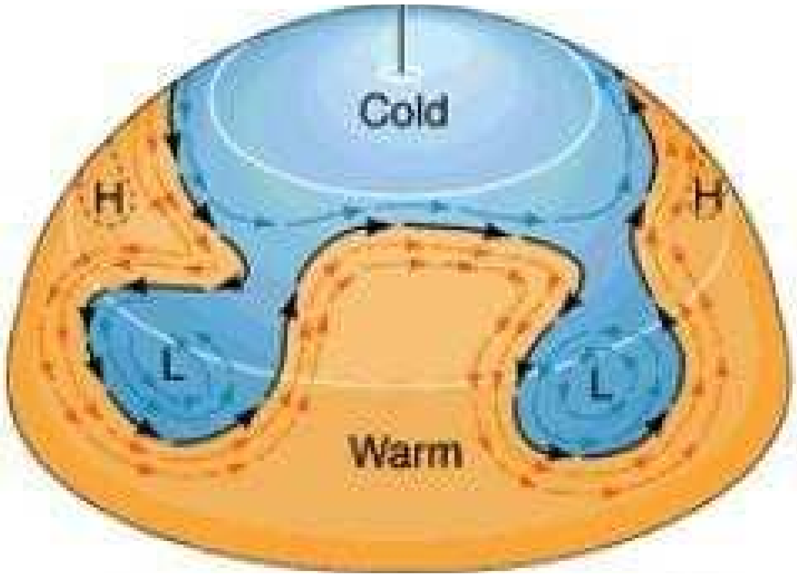
perturbation of jetstream



formation of waves

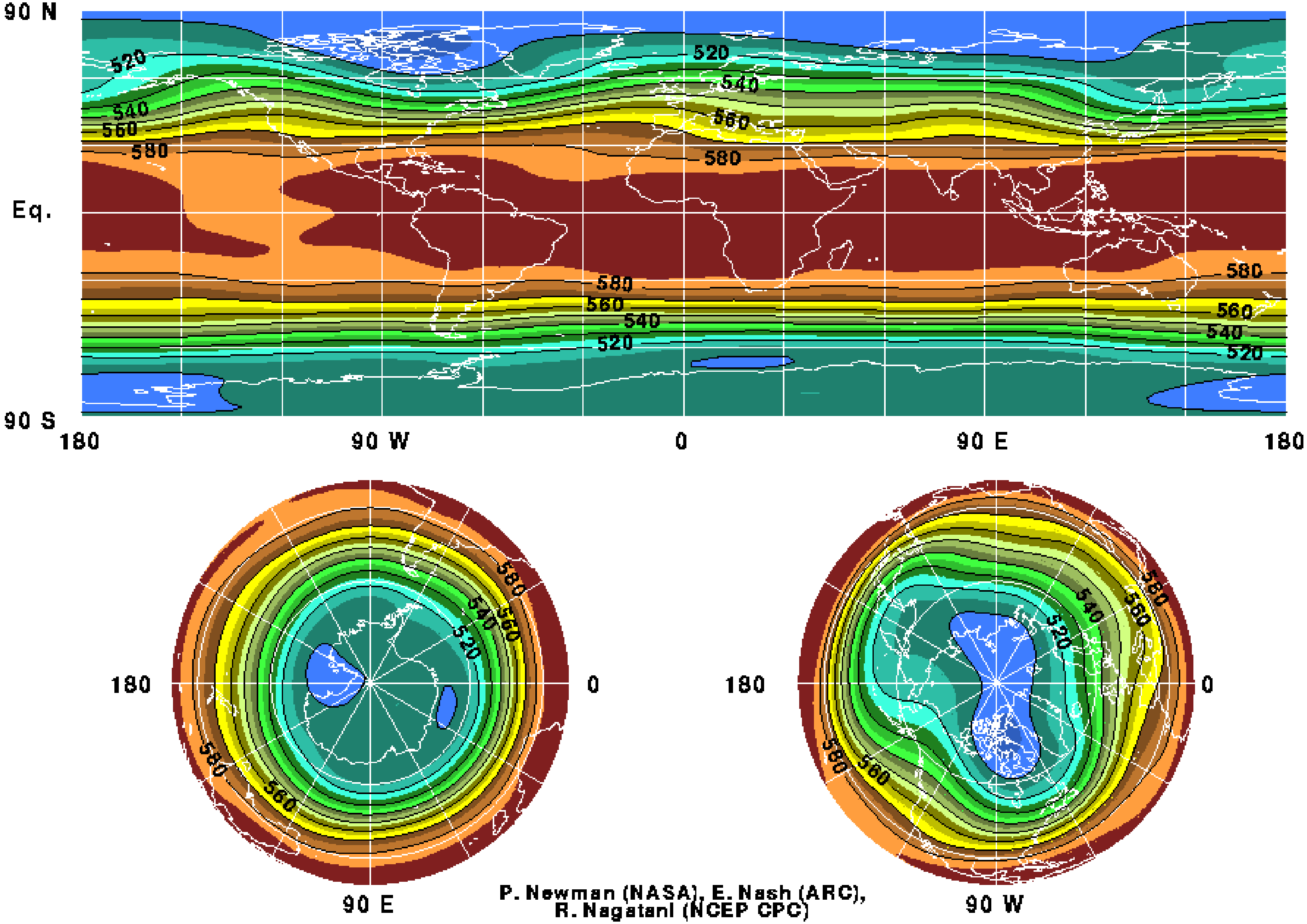


planetary Rossby waves



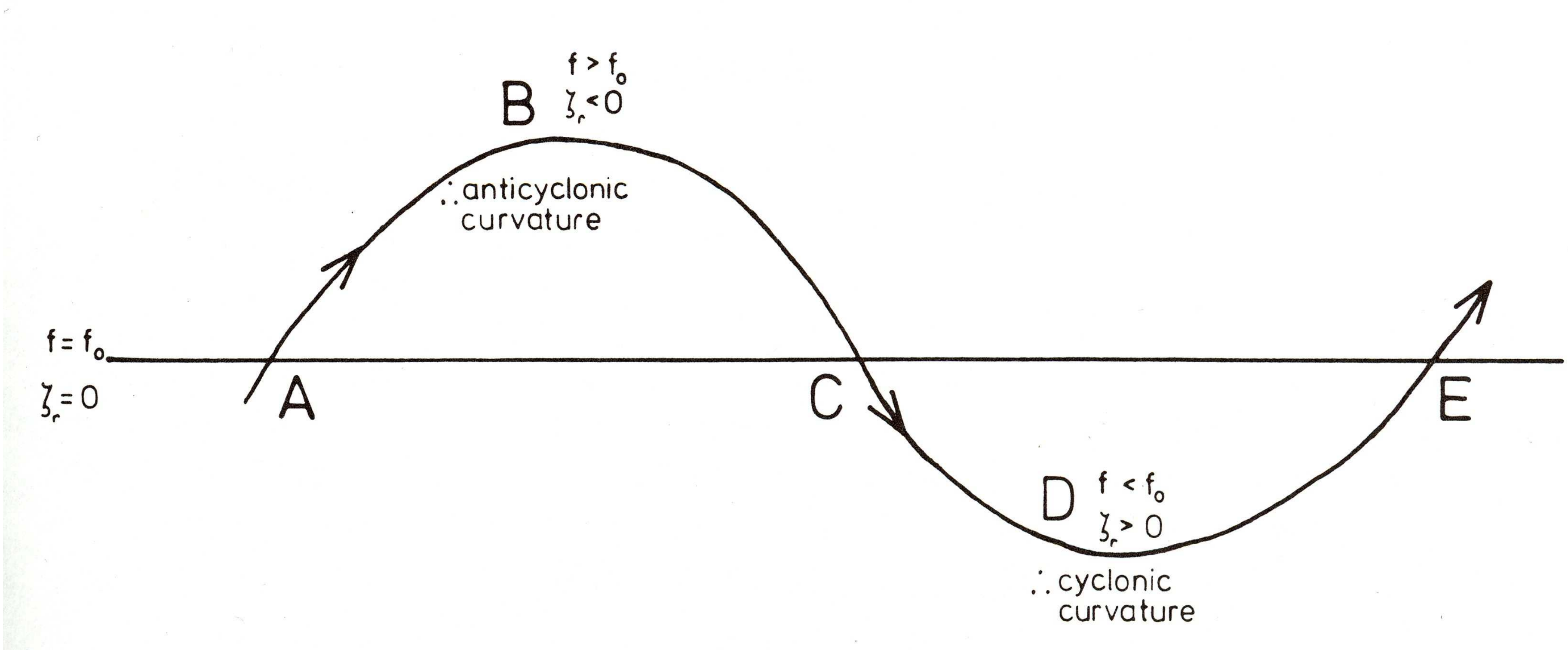
Cut-off process

Geopotential Height (dam)
500 hPa December 1979-1995



P. Newman (NASA), E. Nash (ARC),
R. Nagatani (NCEP CPC)

Trajectory in Long Waves



Neglect shear; vorticity from Curvature alone.

Barotropy

Density function of pressure only

Therefore temperature function of pressure only

Therefore no gradients of T on p surface

Therefore no vertical velocity and no thermal wind.

Vorticity equation

$$\left(\frac{\partial}{\partial t}+u_g\frac{\partial}{\partial x}+v_g\frac{\partial}{\partial y}\right)_p\nabla_p^2\psi+v_g\beta=f_0\frac{\partial\varpi}{\partial p}$$

becomes

$$\left(\frac{\partial}{\partial t}-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\nabla_h^2\psi+\frac{\partial\psi}{\partial x}\beta=0$$

Linearisation

This equation is non-linear

Linearise by considering small perturbations (primes) from basic flow (subscript 0)

$$v_g = (u_0, 0) + (u', v')$$

$$\psi = \psi_0 + \psi' \qquad u_0 = -\frac{\partial \psi_0}{\partial y} \qquad (u', v') = \left(-\frac{\partial \psi'}{\partial y}, \frac{\partial \psi'}{\partial x}\right)$$

$$\left(\frac{\partial}{\partial t} + (u_0 + u')\frac{\partial}{\partial x} + v'\frac{\partial}{\partial y}\right)\nabla_h^2 \psi' + \frac{\partial \psi'}{\partial x}\beta = 0$$

$$\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}\right)\nabla_h^2 \psi' + \frac{\partial \psi'}{\partial x}\beta = 0$$

Solving the linearised equation

Try solutions like $\psi' = A \exp i\{\sigma t + \lambda x + \mu y\}$

Reminder of properties of such solutions

If λ is real, pattern repeats after wavelength $L_x = 2\pi/\lambda$

Likewise for $L_y = 2\pi/\mu$

We need a linear combination of such solutions to achieve

a solution which is real and satisfies any boundary conditions

Phase moves with speeds $c_x = -\sigma/\lambda \quad c_y = -\sigma/\mu$

in x-direction, y direction resp

Group velocity $c_{gx} = -\partial\sigma/\partial\lambda \quad c_{gy} = -\partial\sigma/\partial\nu$

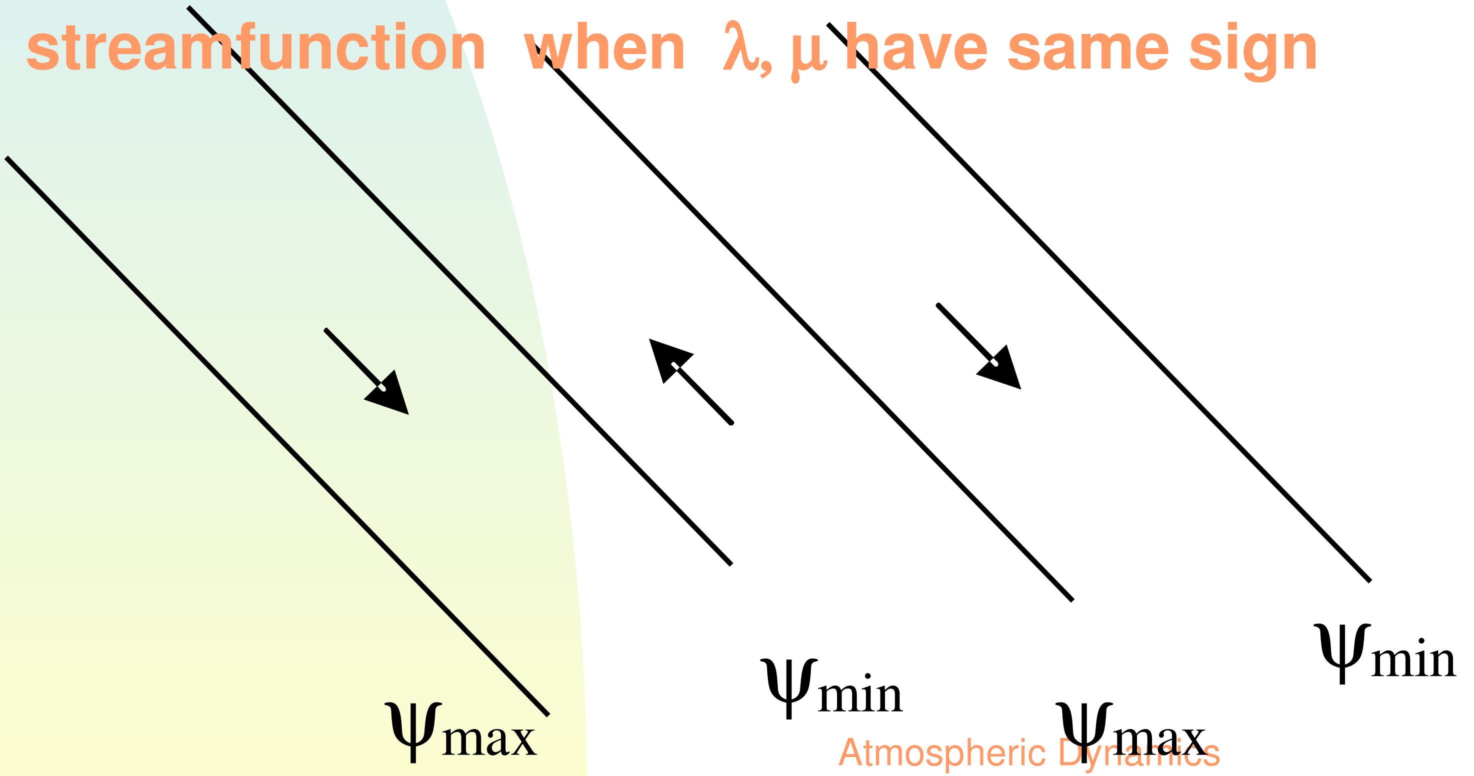
Map of perturbation

magnitude of ψ constant where phase constant

i.e. where $\lambda x+\mu y$=constant

Sketch shows lines of equal perturbation

streamfunction when λ, μ have same sign



Solving the linearised equation

Try solutions like

$$\psi' = A \exp i\{\sigma t + \lambda x + \mu y\}$$

note

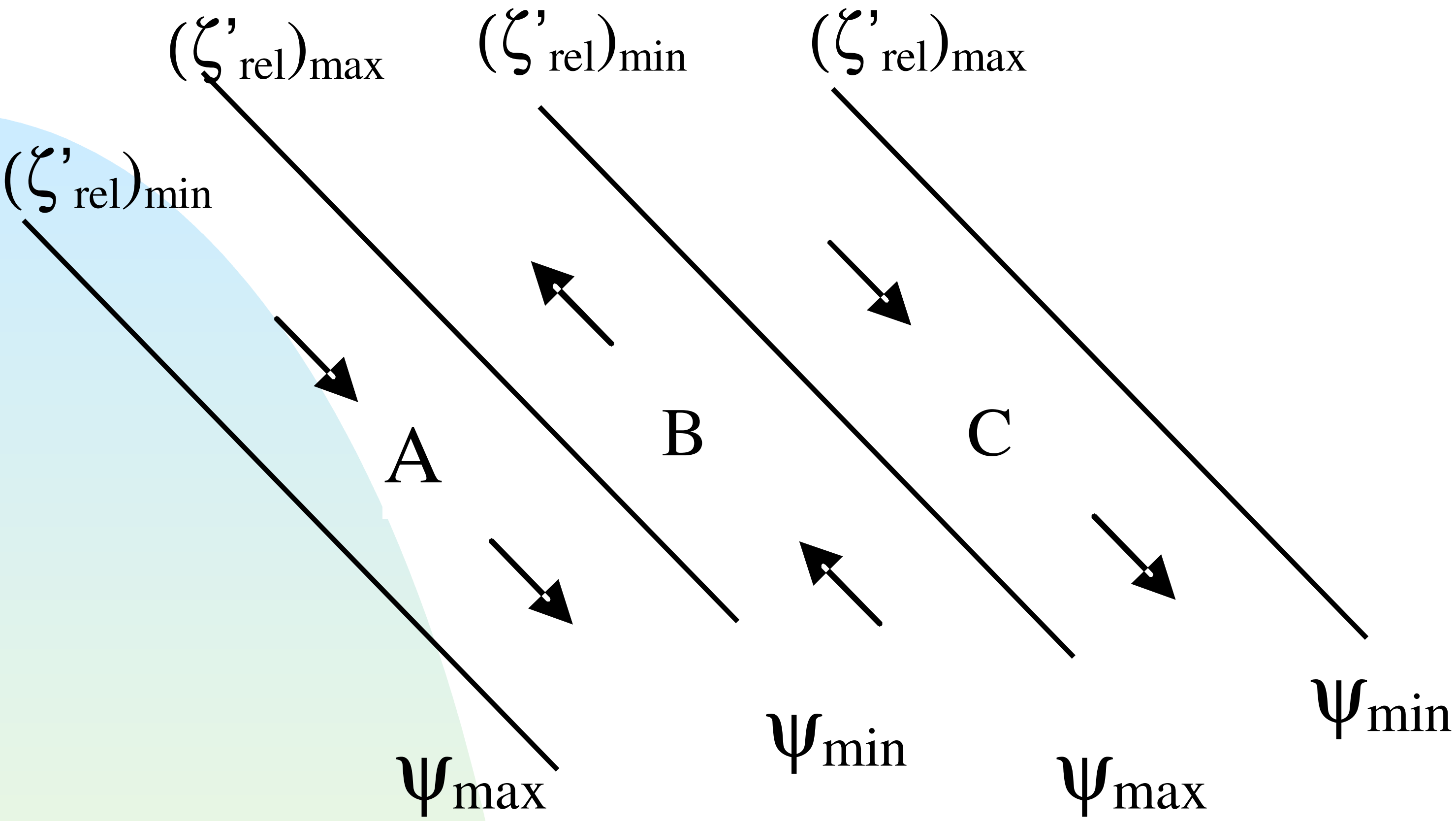
$$\frac{\partial}{\partial t}\psi' = i\sigma A \exp i\{\sigma t + \lambda x + \mu y\} = i\sigma\,\psi'$$

and

$$\frac{\partial}{\partial x}\psi' = i\lambda\,\psi'$$

$$\left(i\sigma + iu_o\lambda\right)\left(-\lambda^2 - \mu^2\right) + i\beta\lambda = 0$$

$$c = -\frac{\sigma}{\lambda} = u_0 - \frac{\beta}{\lambda^2 + \mu^2}$$



Reason for westward phase-speed

At A and C abs vort, and hence

rel vort increases by advection

At B rel vort decreases

Hence lines of max and min rel vort

Size of stationary waves

For c=0

$$\lambda^2 + \mu^2 = \frac{\beta}{u_0}$$

If no N-S variation

$$\mu = 0$$

$$\left(\frac{2\pi}{L_x}\right)^2 = \frac{\beta}{u_0} \longrightarrow L_x = 2\pi\sqrt{\frac{u_o}{\beta}}$$

Basic wind/ (ms^{-1})	*10*	*15*	*20*
Wavelength/km	5960	7300	8430