Problems on Chapter 12: Potential Vorticity

Q 12.1: Consider a marked column of air, the top of which is labelled A, and the bottom labelled B. Suppose this column is forced over a mountain range. Suppose too that, as the column approaches the mountain, the relative vorticity is zero, B is at 1000 hPa and A at 200 hPa (this is typical of the mid-latitude troposphere), and suppose that at the mountain crest A is still at 200 hPa, but that B is raised to where the pressure is 700 hPa. What is the value of the relative vorticity when the column is at the crest? You may neglect variations in the Coriolis parameter and assume that the latitude is 45°N.

Q 12.2: Repeat Q1, but for latitude 60°N.

Q 12.3: Repeat Q1, but take a shorter column for which A is initially at 500 hPa but is raised (in height) to 400 hPa at the crest.

Q 12.4: Suppose that overnight air is sitting at rest over a high, largish landmass at 45°N (this is supposed to remind you rather crudely of Spain), with surface pressure 750 hPa, and that during the day convection heats a circular area of diameter 1000 km of the lower part of airmass and creates large-scale currents such that the lowermost column of air near the surface which was initially 250 hPa thick becomes 300 hPa thick (through the pressure of the middle level air falling more rapidly than the surface pressure. Find the new relative vorticity.

If this relative vorticity is realised as a solid body rotation, what is the windspeed at a distance of 500 km from the centre of the rotation?

Q 12.5: On an f-plane (i.e. assume the tangent plane approximation and take f to be constant) a mountain range, which is of uniform height in the north-south direction, occupies the region $-L \leq x \leq L$, with $L = 500\,\mathrm{km}$. A uniform airstream from the west, with $u = 10\,\mathrm{m\,s^{-1}}$ fills the region $x \leq -L$.

As the airstream crosses the mountain range, marked columns of fluid which are $400\,\mathrm{hPa}$ deep in pressure co-ordinates in the region $x \leq -L$ are observed to have pressure thickness

$$\Delta p = p_0 + (\delta p_0)\left[1 - \cos\left(\frac{x\pi}{L}\right)\right],$$

where $p_0 = 450\,\mathrm{hPa}$ and $\delta p_0 = 25\,\mathrm{hPa}$.

What is the wind at $x = L$?

Supplementary question for discussion

Q 12.6: Question Q 12.4 seems to imply some discontinuities in the meteorological fields. What are they and can they be avoided?