

Atmospheric Dynamics

Lect.12: Potential Vorticity

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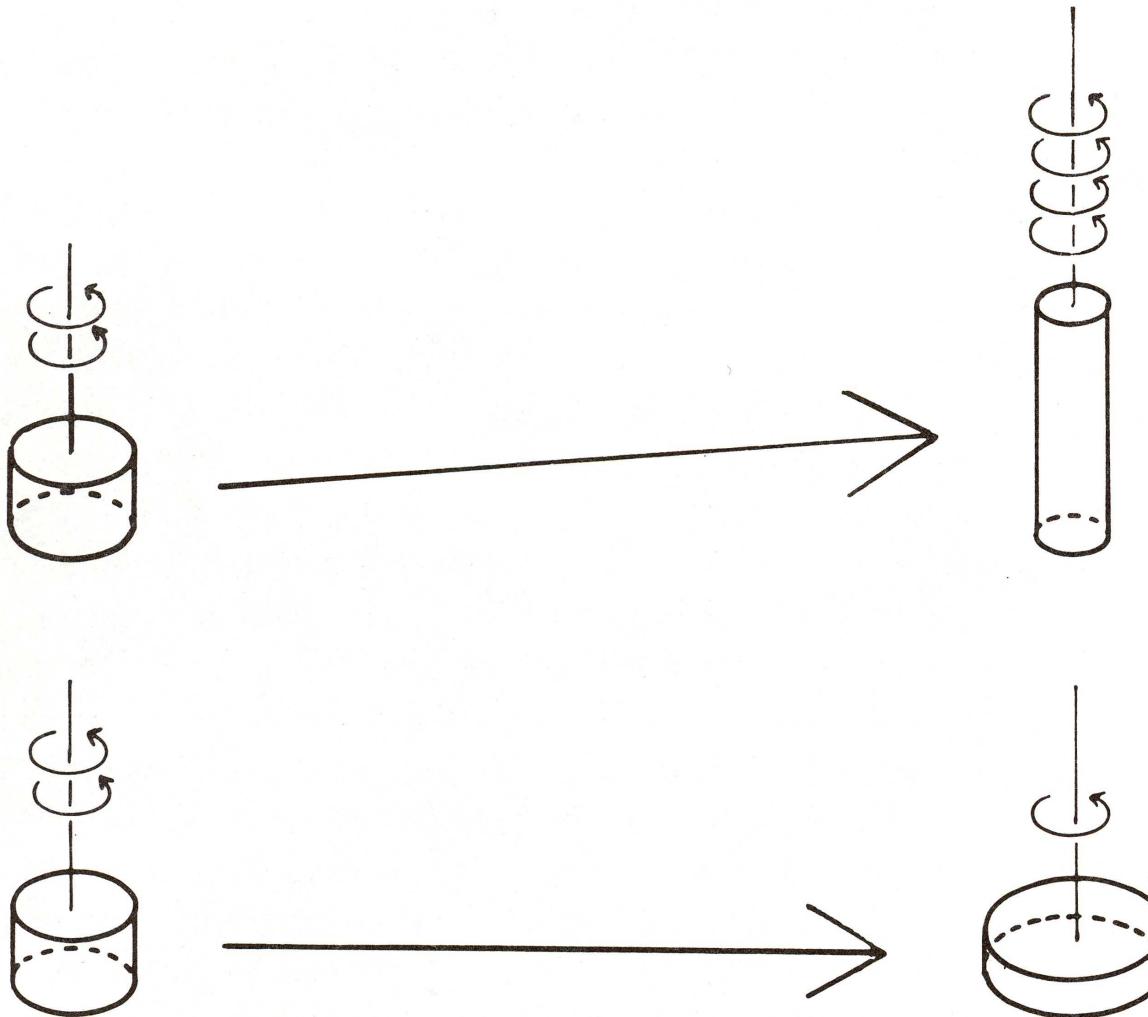
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Crew Building room 218

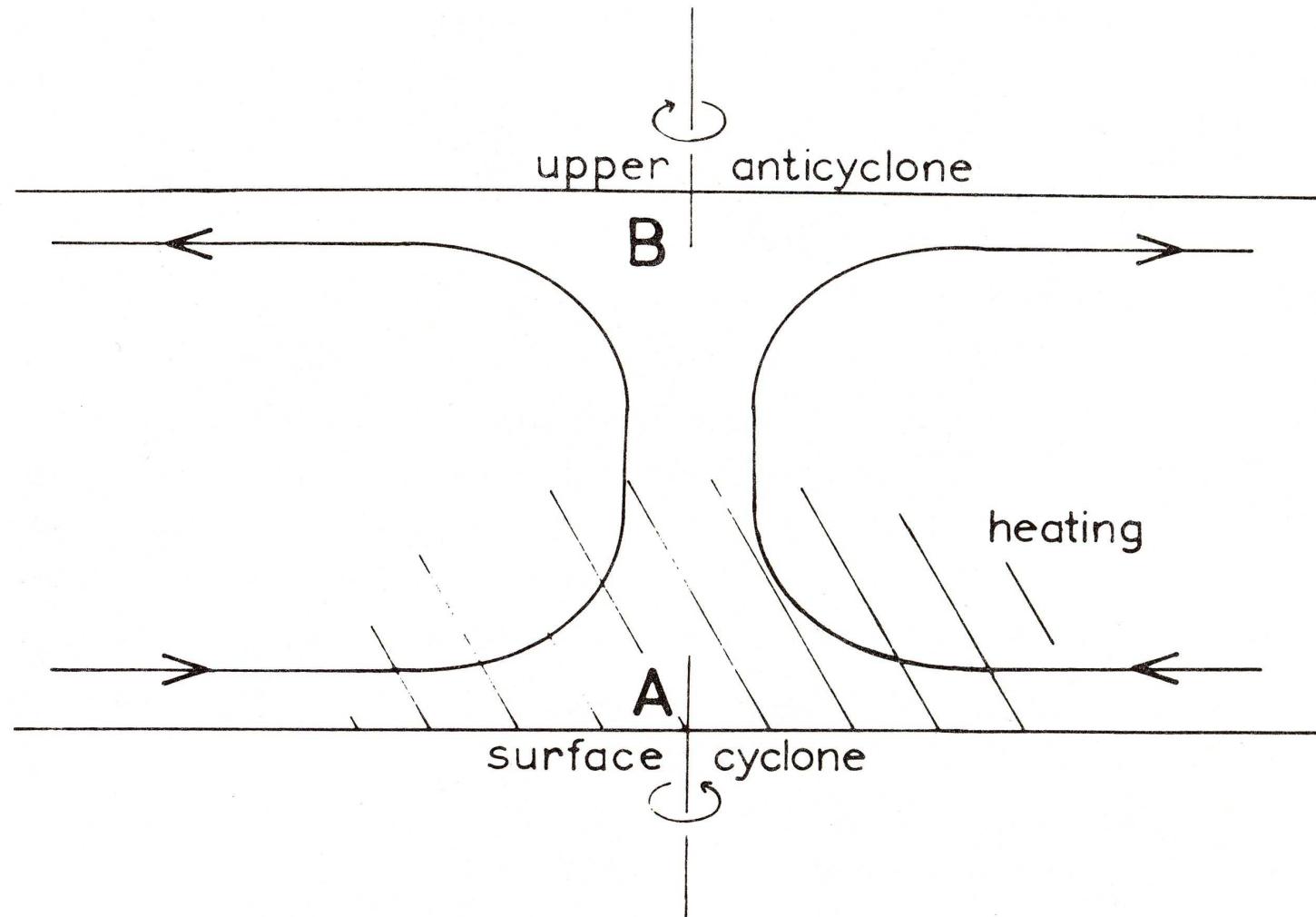
Scope of Lecture

- The heat low
- Limits to vorticity values
- Deflection of flow by a mountain
- Potential vorticity of air column
- Potential vorticity at a point

Vorticity changes



Heat low (mechanism)



Heat low (example)

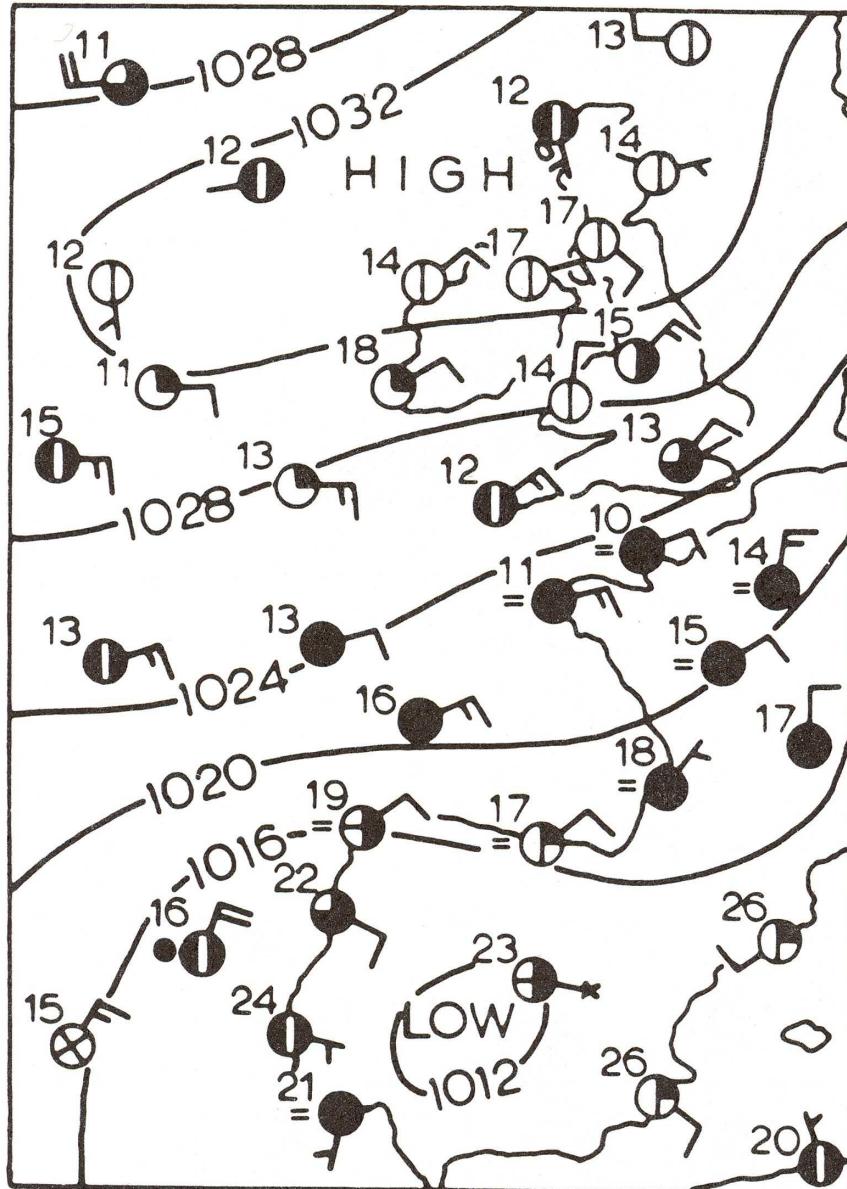


Fig. 4.15 A heat low over Spain.
From *Daily Weather Report* 1200
GMT, 29 May 1963.

Limit to vorticity

The vorticity equation is

$$\left(\frac{D}{Dt} \right)_p \zeta_{abs} = -\zeta_{abs} \operatorname{div}_p \mathbf{v}$$

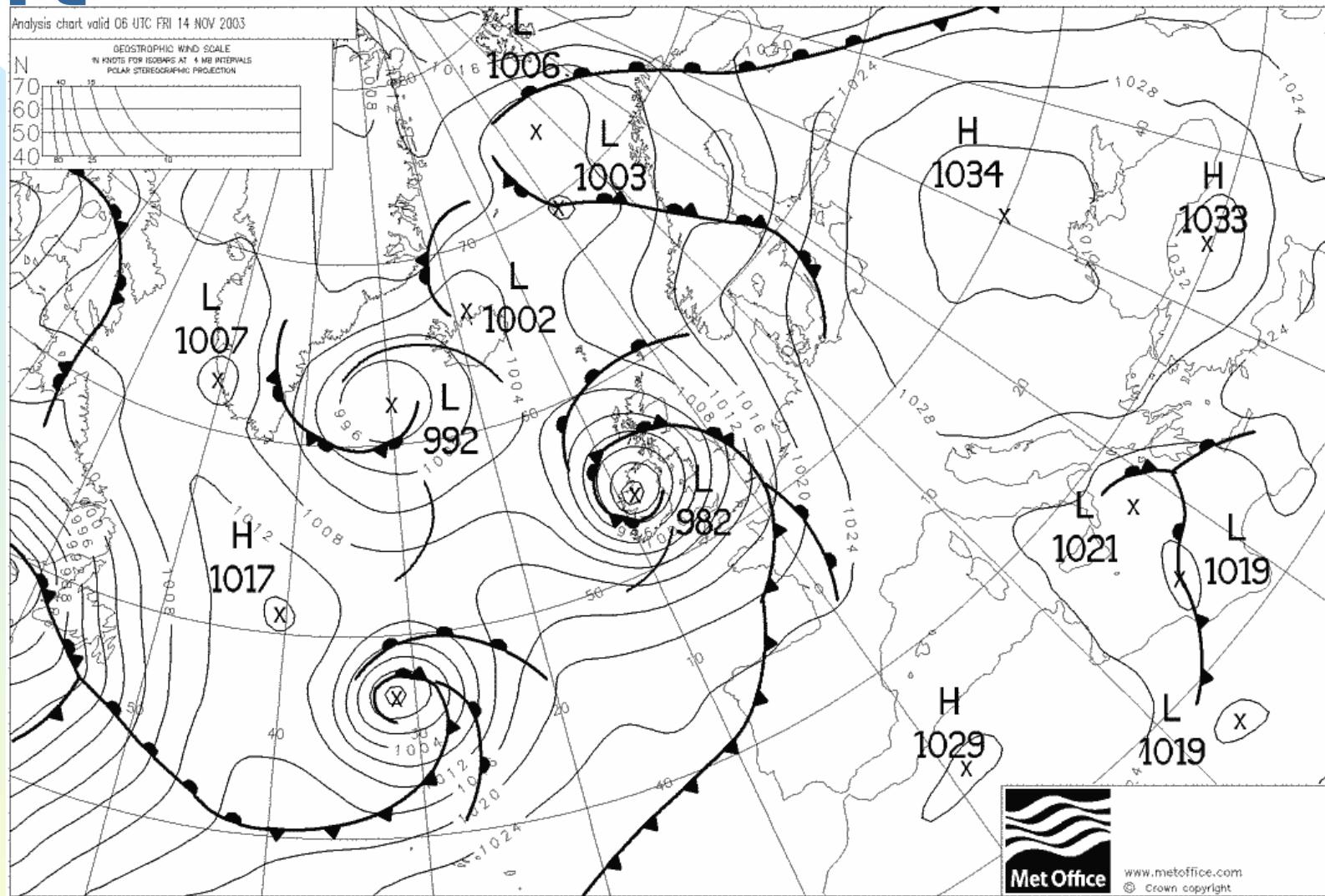
$$\ln \frac{(\zeta_{abs})_2}{(\zeta_{abs})_1} = \int_{t_1}^{t_2} \frac{1}{\zeta_{abs}} \frac{D\zeta_{abs}}{Dt} dt = \int_{t_1}^{t_2} \operatorname{div}_p \mathbf{v} dt$$

Thus sign of absolute vorticity is unaltered

Thus it is always positive

$$-f \leq \zeta_{rel} < +\infty$$

Surface weather chart



Cyclones more vigorous than anticyclones

Potential vorticity

$$\left(\frac{D}{Dt} \right)_p (\zeta_{abs}) = \zeta_{abs} \frac{\partial \bar{\omega}}{\partial p}$$

$$\frac{1}{\delta p} \frac{D\delta p}{Dt} = \frac{\partial \bar{\omega}}{\partial p}$$

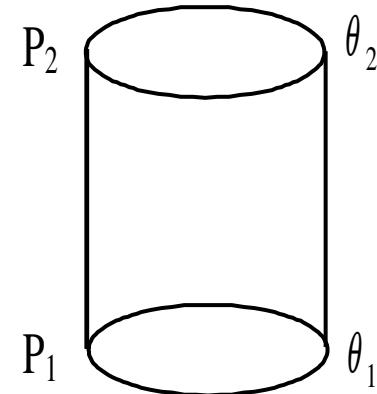
$$\left(\frac{D}{Dt} \right)_p (\zeta_{abs}) - \frac{\zeta_{abs}}{\delta p} \frac{D\delta p}{Dt} = 0$$

$$\frac{\zeta_{abs}}{\delta p}$$

a.k.a. the potential vorticity is conserved

In frictionless flow

Atmospheric Dynamics



$$\delta p = p_1 - p_2$$

$$\left(\frac{D}{Dt} \right)_p \frac{\zeta_{abs}}{\delta p} = 0$$

Flow over mountain

At A $v=0$

$$\frac{\partial u}{\partial y} = 0$$

$$\zeta_{rel} = 0$$

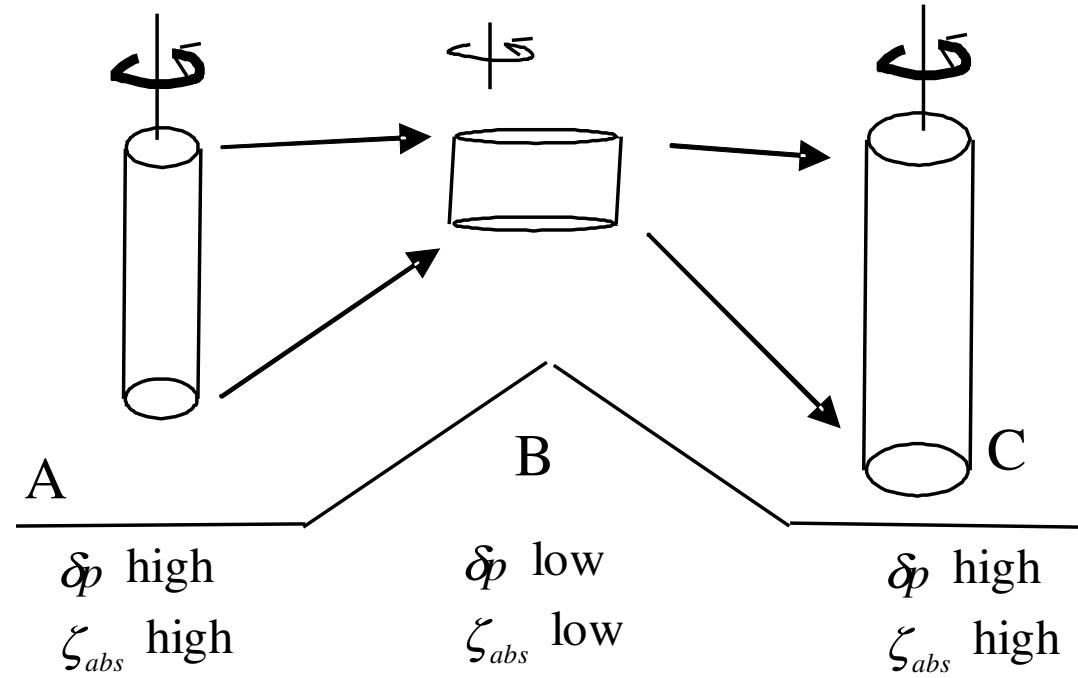
$$\zeta_{abs} = f$$

At B $v=0 \delta p$ small

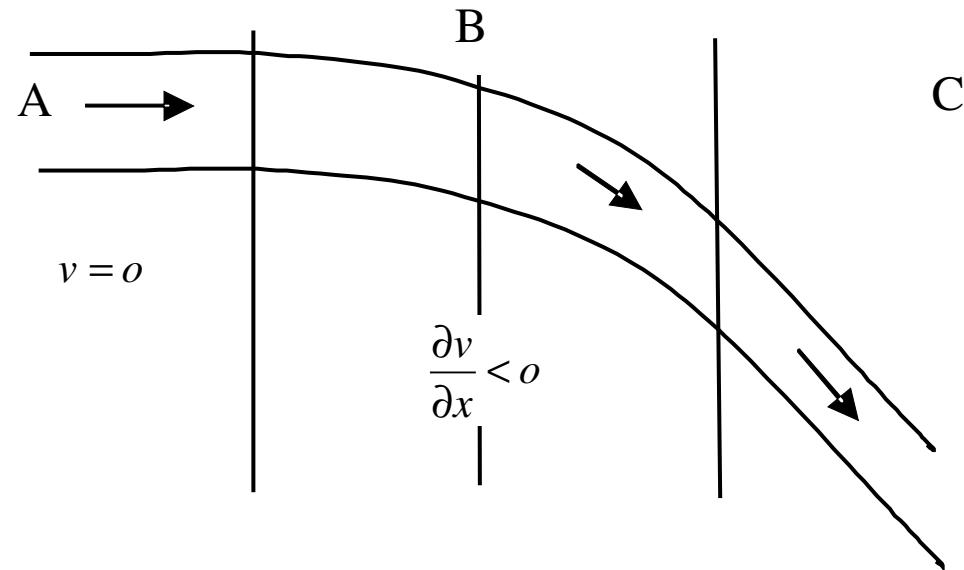
$$\zeta_{abs} < f$$

$$\zeta_{rel} < 0$$

$$\frac{\partial v}{\partial x} < 0$$

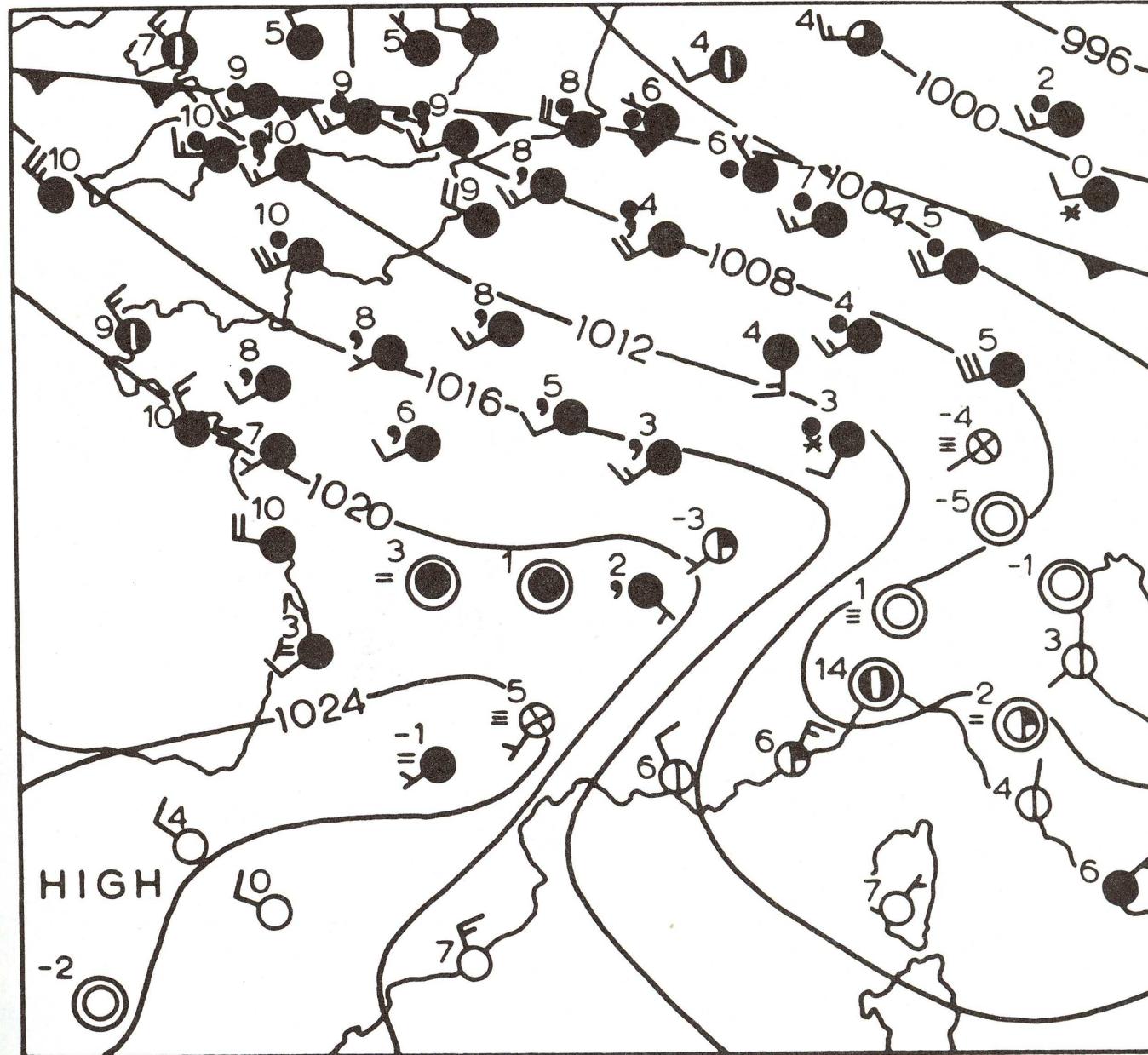


Flow over mountain (cont.)



View from above

Mountain deflection (example)



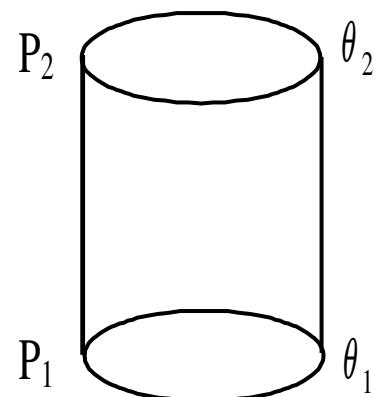
Potential vorticity at a point

$$\frac{1}{p_1 - p_2} \zeta_{abs}$$

$$\frac{\theta_1 - \theta_2}{p_1 - p_2} \zeta_{abs}$$

$$\frac{\partial \theta}{\partial p} \zeta_{abs}$$

$$\frac{1}{\rho} \frac{\partial \theta}{\partial z} \zeta_{abs}$$



Ertel's or isentropic potential vorticity

Full 3-d version:

$$Z = \frac{1}{\rho} (\zeta_{\text{rel}} + \Omega) \nabla \theta$$

when the flow is strongly stable

$$\frac{\partial \theta}{\partial z} \text{ large}$$

=> gradient of pot. Temperature close to vertical

$$Z = \frac{1}{\rho} (\zeta_{\text{rel}} + f) \frac{\partial \theta}{\partial z}$$