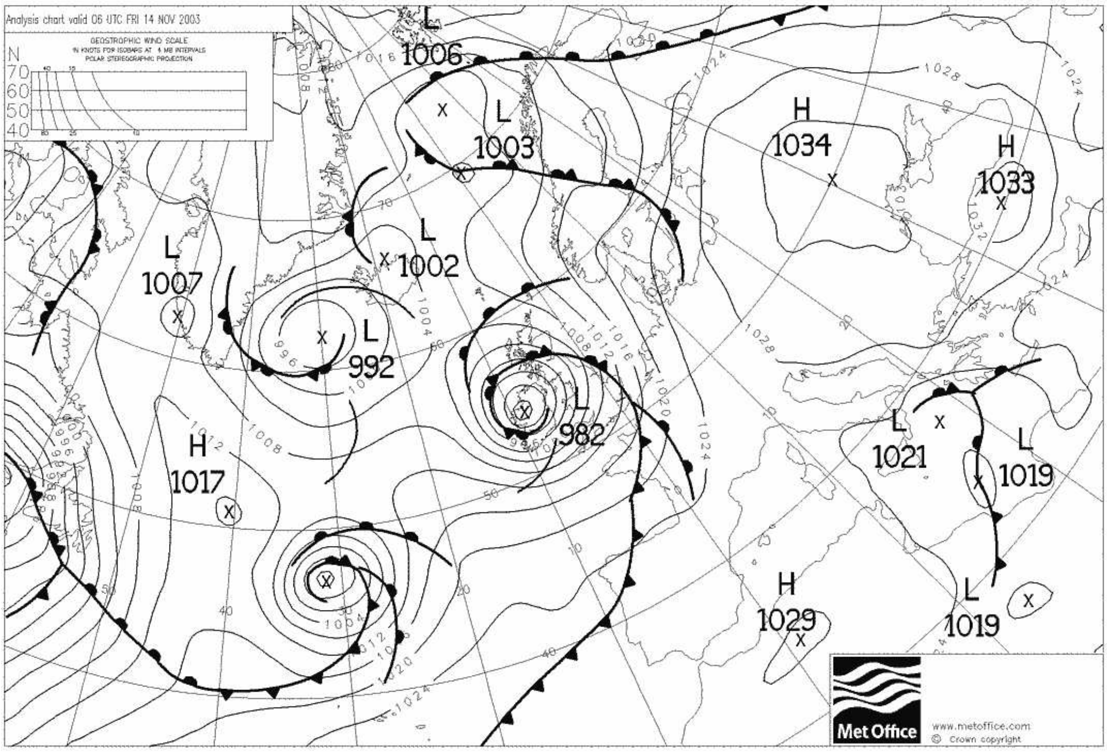
Chapter 12: Potential vorticity: The Heat Low: Deflection of Airflow by Mountains

Limit to anticyclonic vorticity

The vorticity equation in the form we have derived it sets a lower limit on the vorticity but no upper limit. To see this re-arrange the equation as

$$\left(\frac{D}{Dt}\right)_h \ln(\zeta_{abs}) = -div_h \mathbf{v}_h$$

Air which is initially at rest will have positive absolute vorticity. This equation then says how $\ln(\zeta_{abs})$ varies in time, the changes in a finite time simply being given by the time integral of the horizontal convergence. In principle $\ln(\zeta_{abs})$ can reach very large positive or negative values, so that ζ_{abs} may attain very large values, or values very near to zero. However, while the positive values of ζ_{abs} require very large positive relative vorticity, the very small values of ζ_{abs} correspond to $\zeta_{rel} = -f$. Thus the relative vorticity cannot be made more negative than minus the Coriolis parameter, but there is no upper bound to the positive values of relative vorticity which can be attained. This accords with our experience that anticyclones have rather gentle pressure gradients (and hence winds and hence vorticities), whereas pressure gradients at the centre of cyclones can be very large. High surface windspeed are found near the centre of low pressure systems. The difference in vigour of cyclones and anticyclones is clearly visible in the figure below.

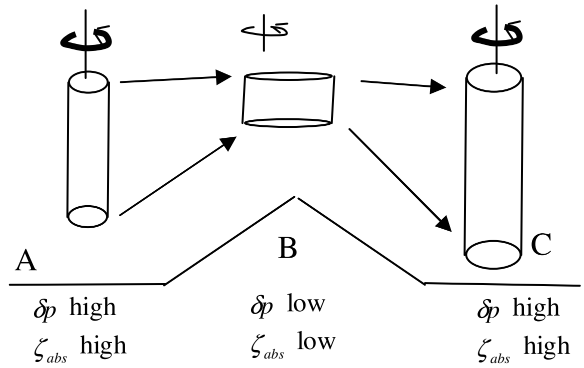


The Heat Low

A description of the Heat Low will be included here. For now see Chapter 4 if Dynamical Meteorology Ed Atkinson Methuen 1981 and the slides associated with this lecture

Deflection of airflows by mountain ranges

We shall consider flow over an isolated mountain range, and for simplicity we will confine our attention to flow which to the west of the range is uniform and from west to east. We shall also assume no variation in the north-south direction

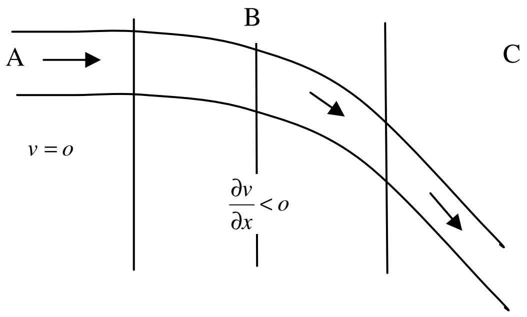


Air at A has $v = 0$ and $\frac{\partial u}{\partial y} = 0$.

δp at B is smaller than at A, thus ζ_{abs} at B is also smaller than at A, so that the potential vorticity remains constant.

Now we have chosen conditions at A such that $\zeta_{rel} = 0$ there, so that at A $\zeta_{abs} = f$. It follows that at B $\zeta_{abs} < f$ and hence that $\zeta_{rel} < 0$ there.

This argument shows that $\zeta_{rel} < 0$ all the time that the air is over the mountain. Since we have required no variation in the north-south directions $\frac{\partial u}{\partial y} = 0$, so that we must have $\frac{\partial v}{\partial x} < 0$. Thus v gets progressively more negative as the air flows across the mountain.



View from above

Hence mountains deflect uniform westerly airstreams towards the South.

Potential vorticity

There is a useful dynamical quantity which is conserved by fluid particles under certain ideal conditions. Our starting point will be **Error! Reference source not found.**a. The basic idea is that the final term factor on the right is to do with air columns getting longer (in pressure co-ordinates), so if we divide the absolute vorticity by some appropriate function of the length of an air column we will end up with something which stays constant.

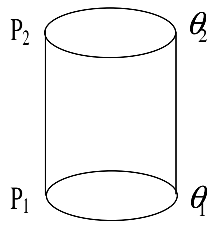


Figure 1

Consider the motion of a column of fluid which we will mark somehow (say with smoke) and which has its top at pressure p_2 where the potential temperature is θ_2 and its bottom at pressure p_1 where the potential temperature is θ_1. As the column moves about, p_1 and p_2 will change. If the motion is adiabatic θ_1 and θ_2 will not change, but if it is diabatic then they will.

Now just as we had $\frac{1}{\delta A}\frac{D\delta A}{Dt} = div_h \mathbf{v}_h$ so we can show that, for a marked column of length $\delta p = p_1 - p_2$, that there is an analogous relation $\frac{1}{\delta p}\frac{D\delta p}{Dt} = \frac{\partial \varpi}{\partial p}$, which is correct to first order in small quantities. Inserting this into **Error! Reference source not found.** gives

$$\left(\frac{D}{Dt}\right)_p (\zeta_{abs}) - \frac{\zeta_{abs}}{\delta p}\frac{D\delta p}{Dt} = 0$$

Dividing by δp we see that this implies that

$$\boxed{\left(\frac{D}{Dt}\right)_p \frac{\zeta_{abs}}{\delta p} = 0}$$

That is $\frac{\zeta_{abs}}{\delta p}$ is constant for the air column. It is sometimes known as *potential vorticity*, but we shall derive a more fundamental version shortly.

Note that we have implicitly assumed that the motion is frictionless and large-scale (needed for the approximations inherent in our form of the vorticity equation).

Potential vorticity at a point

The version of potential vorticity described above is for a finite column of fluid. Such columns eventually deform, so it would be preferable to define a potential vorticity which is a point quantity.

We can obtain this by referring to Figure 1. Since p_1 and p_2 are the ends of a marked column of air, the values of θ_1 and θ_2 will stay the same under adiabatic flow. Thus since $\frac{1}{p_1 - p_2}\zeta_{abs}$ so is $\frac{\theta_1 - \theta_2}{p_1 - p_2}\zeta_{abs}$. In the limit where $p_1 \to p_2$, we see that

$$\boxed{\frac{\partial \theta}{\partial p}\zeta_{abs} \text{ is conserved for frictionless, adiabatic motion.}}$$

This quantity is a version of potential vorticity defined for points in the fluid. In z co-ordinates it looks like $\frac{1}{\rho}\frac{\partial \theta}{\partial z}\zeta_{abs}$.

There is a version of potential vorticity known as Ertel's potential vorticity after the scientist who discovered it, which is true adiabatic, frictionless fluids of all scales. This quantity is $Z = \frac{1}{\rho}(\boldsymbol{\zeta}_{rel} + \mathbf{\Omega})\nabla\theta$. Note that in this expression the vector quantities (namely relative vorticity, Earth's rotation and the gradient of potential temperature) are the full 3-D versions. Proving this relationship is beyond the scope of our course. Ertel's potential vorticity reduces to the form we have deduced for large scale flow when the flow is so strongly stable ($\frac{\partial \theta}{\partial z}$ large) that the gradient of potential temperature is so close to the vertical that the terms from the horizontal components of the vorticity become negligible in comparison.