Atmospheric Dynamics Lect.11: Rates of change of Vorticity and Divergence

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Scope of Lecture

- Continuity Equation
  - relation between w and div**v**
- Magnitude of w
- Distributions of convergence and divergence with middle level vertical velocity
- The divergence equation
- Pressure co-ordinate versions

Continuity equation

For element, density x vol=const --> $$\frac{D}{Dt}(\rho\delta\tau)=0$$

--> $$\frac{D}{Dt}\ln(\rho\delta\tau)=0 \qquad \frac{1}{\rho}\frac{D\rho}{Dt}+\frac{1}{\delta\tau}\frac{D\delta\tau}{Dt}=0$$

but $$\frac{1}{\delta\tau}\frac{D\delta\tau}{Dt}=div\mathbf{v}$$

$$\frac{1}{\rho}\left\{\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z}\right\}+div\mathbf{v}=0$$

Forms of the continuity equation

$$\frac{1}{\rho}\left\{\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}\right\}+div\mathbf{v}=0$$

$$\frac{D}{Dt}\rho+\rho div\mathbf{v}=0$$

Lagrangian form

$$\frac{\partial \rho}{\partial t}+div\left(\rho\mathbf{v}\right)=0$$

Eulerian form

Approximate form

We had

$$\frac{1}{\rho}\left\{\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}\right\}+div\mathbf{v}=0$$

regroup

$$\left\{\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}\right\}+\rho div_h\mathbf{v}_h+\rho\frac{\partial w}{\partial z}=0$$

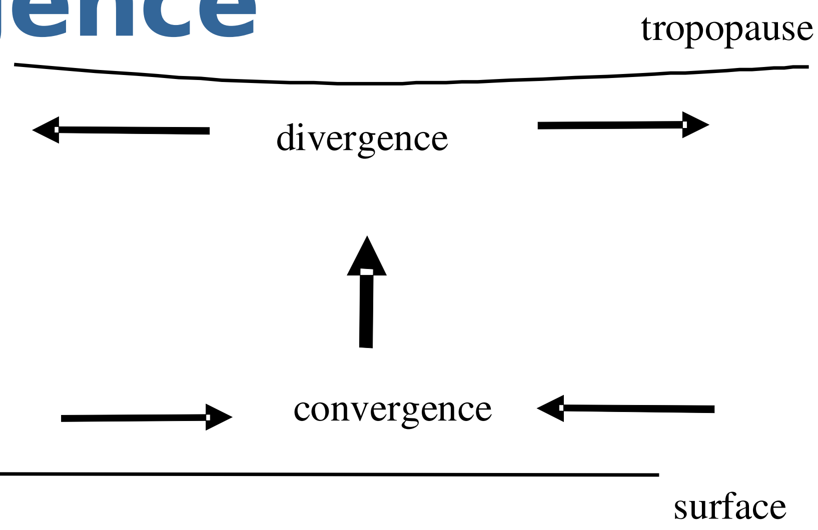
$$\left\{\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}\right\}+\rho div_h\mathbf{v}_h+\frac{\partial (w\rho)}{\partial z}=0$$

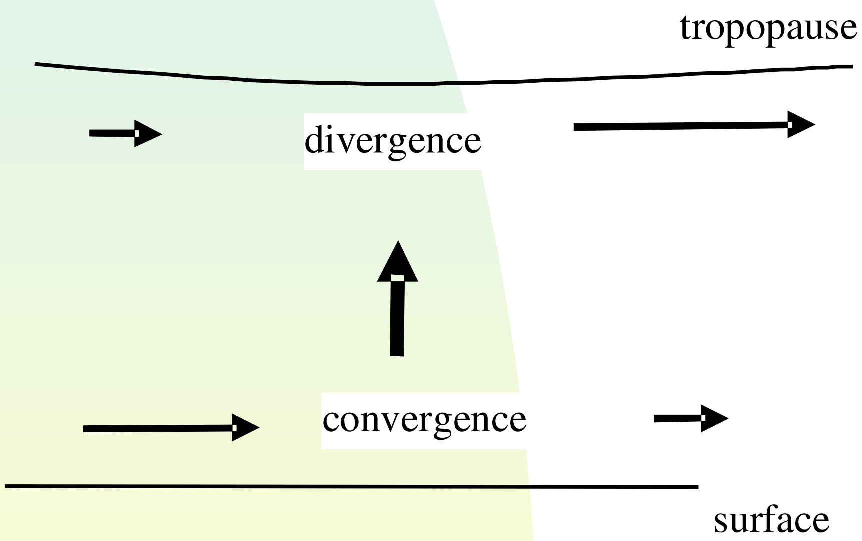
Curly bracket negligible

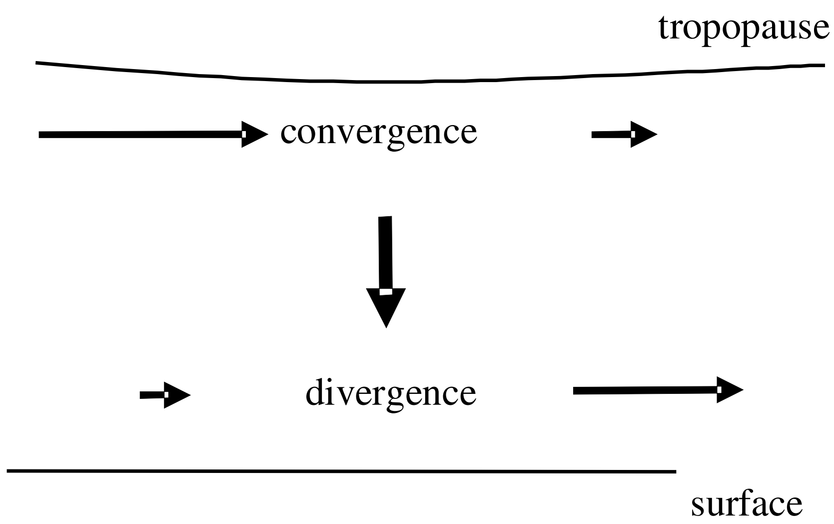
$$\rho div_h\mathbf{v}_h+\frac{\partial (w\rho)}{\partial z}=0$$

Some configurations of vertical velocity and divergence

$$\rho \, div_h \mathbf{v}_h + \frac{\partial (w\rho)}{\partial z} = 0$$







Magnitude of vertical velocity

$$\rho\, div_h \mathbf{v}_h + \frac{\partial (w\rho)}{\partial z} = 0 \qquad \frac{\partial w}{\partial z} + \frac{w}{\rho}\frac{\partial \rho}{\partial z} = -div_h \mathbf{v}$$

Introduce scales L,H,U,W

LHS of order W/H RSH of order R_o (U/L)

W~R_oU (H/L)

W~(1/10) x (10ms^{-1})(10km/1000km)

= 1 cm s^{-1} ~ 1 km day^{-1}

Approximate material derivative

$$\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$U / L$$

$$\frac{W}{H} = \frac{R_o UH}{HL} = R_o \frac{U}{L}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + O\left(R_o\right)$$

Vorticity Equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv=-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)+\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial u}{\partial y}\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\frac{\partial u}{\partial y}+f\frac{\partial u}{\partial x}+f\frac{\partial v}{\partial y}+v\frac{\partial f}{\partial y}$$

$$=-\frac{1}{\rho}\frac{\partial^2 p}{\partial x\partial y}+\frac{1}{\rho}\frac{\partial^2 p}{\partial y\partial x}$$

$$\left(\frac{D}{Dt}\right)_h\left(\zeta_{rel}+f\right)=-\left(\zeta_{rel}+f\right)div_h\mathbf{v}_h$$

$$\left(\frac{D}{Dt}\right)_h\left(\zeta_{abs}\right)=-\left(\zeta_{abs}\right)div_h\mathbf{v}$$

Continuity eq in p-co-ordinates

In z co-ords we had

$$\rho div_h \mathbf{v}_h + \frac{\partial(w\rho)}{\partial z} = 0$$

$$div_p \mathbf{v}_h + \frac{\partial \varpi}{\partial p} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \varpi}{\partial p} = 0$$

Vorticity equations in p-coordinates

$$\left(\frac{D}{Dt}\right)_h(\zeta_{abs}) = \frac{\zeta_{abs}}{\rho}\frac{\partial(\rho w)}{\partial z}$$

$$\left(\frac{D}{Dt}\right)_p(\zeta_{abs}) = \zeta_{abs}\frac{\partial\varpi}{\partial p}$$

$$\left(\frac{D}{Dt}\right)_p(\zeta_{abs}) = -\zeta_{abs}div_p\mathbf{v}_h$$