Problems on Chapter 10: Vorticity and Divergence

Q 10.1: On a particular occasion the wind was $14.14\,\mathrm{m\,s^{-1}}$ from the NW near the surface and $14.14\,\mathrm{m\,s^{-1}}$ from the SW at a height of 4 km. What were the horizontal components of relative vorticity?

Q 10.2: On a particular occasion the wind in the vicinity of a point O was observed to be blowing from SE to NW in straight parallel lines with windspeed increasing towards the NE by $20\,\mathrm{m\,s^{-1}}$ for each 500 km. What was the relative vorticity? If this was at 45°N, what was the absolute vorticity?

Q 10.3: In the lecture and notes we used the relation

$$\mathrm{div_h}\,\boldsymbol{v}_h = \frac{1}{\delta A}\frac{D}{Dt}\delta A$$

where δA is the area of a horizontal area of small marked element of fluid which is initially square. Prove this.

Q 10.4: Consider a horizontal velocity $\boldsymbol{v}_h$ given by $\boldsymbol{v}_h = \boldsymbol{v}_r + \boldsymbol{v}_d$, with the terms on the right hand side of the form $\boldsymbol{v}_r = k \times \nabla \Psi$ and $\boldsymbol{v}_d = \nabla \Phi$. Show that $\boldsymbol{v}_r$ has vorticity but is non-divergent, and that $\boldsymbol{v}_d$ has divergence but is irrotational (i.e. has no vorticity). Show too that Ψ and Φ (known as the stream function and velocity potential respectively) can always be found by solving an appropriate equation involving the Laplacian, so that the wind can always be split into a rotational and divergent part.

(Hint: take div and curl of the velocity)