# **Chapter 9: Tropical circulations**

Given the complexity of the motion systems in the atmosphere, some fluid dynamicists have performed experiments in simple fluid systems in the laboratory in the hope that the simpler, more controllable behaviour that results may offer insight which can then be applied to the complexity of the real world. Some of these experiments suggest that the tropical regions may behave significantly differently from extra-tropical regions. A typical experiment involves a rotating turntable with several concentric cylinders.

The slides which accompany this lecture (see web page) shows a typical apparatus. It consists of 3 concentric hollow cylinders attached by waterproof seals to a flat circular base, with their axes vertical. Let us label them A, B and C from the innermost to the outermost. The working fluid which represents the atmosphere is placed between cylinder A and B. Cylinder A is filled with a liquid which is kept at a constant temperature, and the space between cylinder B and C is filled with a liquid which is kept at different constant temperature. If A contains the warmer liquid, then the working liquid gets warmed at the wall of cylinder A and cooled at the cylinder B. The whole apparatus can be rotated, and it is possible to view the apparatus in a frame of reference which rotates with it. The fact that the whole thing is mounted on a rotating turntable is an analogy to the vertical component of the earth's rotation.

For a given temperature difference between the constant temperature baths the behaviour of the working fluid is observed to depend on the rotation rate. For zero rotation fluid rises up the warm wall and travels radially across to the cold wall where it sinks, with a return flow to the warm wall near to the bottom. When a modest rotation is introduced, the flow (seen in a frame of reference rotating with the apparatus) remains axially symmetric, but is no longer in the radial direction, as zonal (tangential) motions are induced by the Coriolis forces acting on the radial components of the particle's motion.

When the rotation rate is increased further, regular wavy motion is produced. Some movie clips showing the motions for various rotation rates and temperature contrasts have been placed on the website for the lecture series. As the rotation rate is increased further, the number of waves increases and they become more variable, with the amplitude growing and decaying, initially with a regular period. Further increase in the rotation rate gives even more waves and the onset of a more chaotic behaviour in time, until eventually the motion is very irregular. It is found that for some combinations of rotation and temperature difference the behaviour is quite reproducible.

As the vertical component of the Earth's rotation is low in the tropics, and high in middle and high latitudes, these experiments suggest that tropical motions might be zonally symmetric, but that mid-latitude flow will be wavy or chaotic. Observations of the atmosphere broadly support this contention. Although some longitudinal variations are found in the tropics, the variations are much smaller than those in middle and high latitudes. Because of this it is profitable to consider zonally symmetric motions as a first step to understanding some of the phenomena observed in low latitudes.

## Axi-symmetric motion

If the flow does not vary with longitude, a marked ring of fluid particles (imagine it full of smoke so that we can keep track of which ring we mean) initially lying all the way around a latitude circle at a fixed height will remain ring-shaped and remain aligned with the latitude circles. If there are no frictional forces to exert a torque the angular momentum of this ring about the earth's axis of rotation will stay constant. Thus if r is the distance of the ring from the axis of rotation and  $\varpi$  is its rate of rotation seen from inertial space and if its mass is m then it will move so that  $\varpi mr^2$  is constant, which as mass is conserved means that

$$\varpi r^2 = const$$

Now the total angular velocity is the sum of the angular velocity of the earth's rotation, plus the angular velocity of the air motion relative to the earth

$$\varpi = \Omega + \frac{u}{r}$$

But  $r = (a + z)\cos\phi$  giving

$$\left\lfloor \Omega + \frac{u}{(a+z)\cos\phi} \right\rfloor (a+z)^2 \cos^2\phi = const$$

If we know the zonal velocity, u, at one latitude and height, this will allow us to find the zonal velocity which the air will attain at some other latitude and height. This equation can be simplified considerably by noting that a is in excess of 6000km, while z is less than 20km in the troposphere and is unlikely to exceed 100km in any meteorological application, so to good approximation we may neglect z in comparison with a to give

$$\Omega a \cos^2 \phi + u \cos \phi = const$$

Eq 2

Eq 1

#### Equinox case

Consider the equinox case when the sun is overhead at the equator. Sunlight is absorbed at the ground which heats up, warming the air in contact with it. This air rises and spreads out polewards. At low levels at the equator the zonal wind is small due to friction with the ground, so we will assume u = 0 at  $\phi = 0$ . Hence subsequent motion has

$$\Omega a \cos^2 \phi + u \cos \phi = \Omega a$$

or

$$u = \Omega a (1 - \cos^2 \phi) / \cos \phi$$

Thus we expect strong westerly winds to develop as this ring of air leaves the ground near the equator and move at high levels to the subtropics. The development of the westerly winds can also be regarded as the effect of the Coriolis force acting on the poleward component of the motion. This explains the existence of the subtropical jets at around 30N and 30S.

### Solstice case

In this case the hottest places at the surface are off the equator following roughly the latitude at which the sun is overhead at noon. Thus the rising motion is at this latitude,  $\phi_a$  say, and it is at this latitude that we should set u = 0. Thus subsequently

$$\Omega a \cos^2 \phi + u \cos \phi = \Omega a \cos^2 \phi_a$$

Eq 3

What happens at the equator in this case? Using Eq 3 gives the wind at the equator as

$$u = \Omega a \left( 1 - \cos^2 \phi_o \right) < 0$$

Thus the winds aloft at the equator can be expected to be easterly.



The figure above shows the zonally averaged flow in the form of a stream function. That is, the flow in the meridional plane, when averaged along lines of equal latitude and height, is parallel to the contours on the figure and inversely proportional to their spacing. The flow in equatorial regions is dominated by the Hadley cell which has upward flow in the summer hemisphere and downward flow in the winter hemisphere. There is another weaker branch of the Hadley cell which has the descent in the summer hemisphere, but much weaker than the descent to be found in the winter hemisphere. Rings of air are carried in this flow. The analysis we have just performed implies that there will be easterlies above the equator and westerlies at the polemost extreme of the two branches of the Hadley cell.



Zonal mean zonal wind component

The figure above shows the zonally averaged zonal wind component.

# Asymmetries

The previous discussion was based on the idea that the flow is zonally uniform. That is only a first approximation. There are a couple of notable phenomena for which zonal symmetry is not relevant, but we do not have time to go into details in this course.

One is the circulations associated with the El Nino phenomena. This occurs somewhat irregularly every few (4-5) years. The sea temperatures in the eastern tropical Pacific become unusually warm, and air rises preferentially in that region and sinks elsewhere in the same latitude band. Because some aspects of this were first noted as a oscillation in the pressure difference between Darwin (Australia) and Tahiti, which was initially christened "the Southern Oscillation", the whole process is now often called ENSO, standing for El-Nino-Southern-Oscillation.

Another remarkable feature of tropical circulations is that in the tropical stratosphere the zonal winds undergo an oscillation with a period of approximately two years, with easterly winds one year and westerlies the next. The actual period varies somewhat from one event to the next, with an overall average period of about 27 months. One interesting feature of this is that in the westerly phase the angular momentum in the westerlies is higher than anywhere else on the planet, raising the question of how the westerly momentum can be supplied to that region, since we have already seen earlier in this chapter that theories of conservation of angular momentum can only produce easterlies. There is a pretty good theory of how this occurs based on momentum fluxes due to gravity waves (and other waves) regarded as a sort of eddy viscosity. However it is a little misleading to call it "viscosity" as the flux is clearly up the gradient of momentum towards high values.